2.5 and 2.6. Linear Functions, Linear Equations and their graphs

Tuesday, September 11, 2018 12:56 PM

Objectives: (1) Understand the equation $y = mx + b$ or

$$f(x) = mx + b$$

(2) Given 2 points on a line, find the slope of the line.
Given a linear equation, derive its slope-intercept form.

(3) Solve some applications

(4) Graph linear functions
- using intercepts
- using slope and y-intercept

(5) Vertical Lines and Horizontal Lines

(6) Parallel and Perpendicular Lines

(1) A **linear** function is a function of the form

$f(x) = mx + b$ or $y = mx + b$. (m, b: constants)

This is called the slope-intercept form.

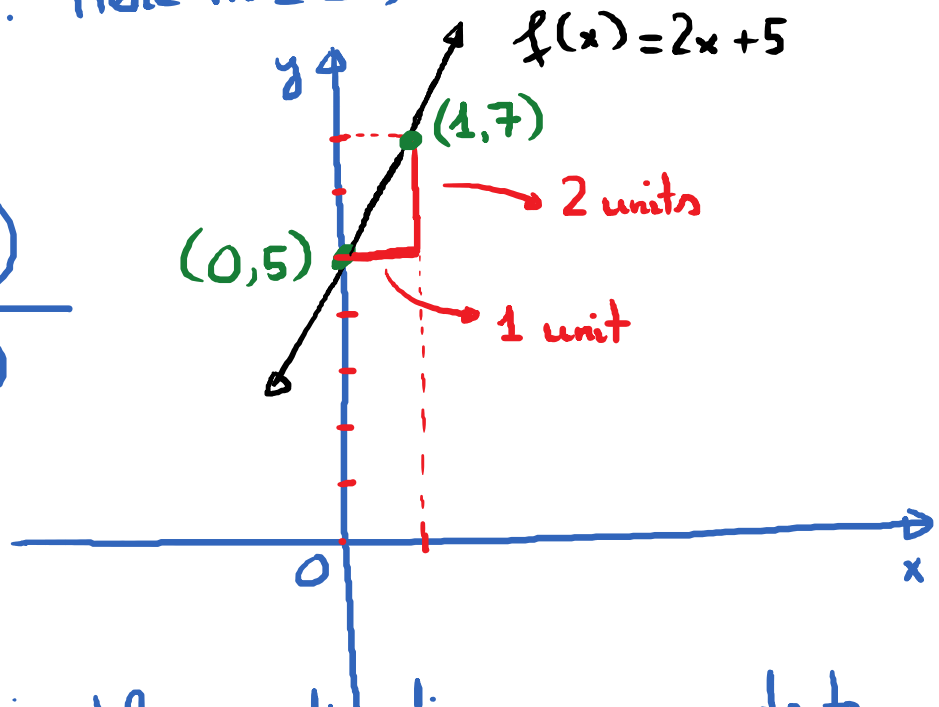
"Linear" means the graph is a straight line

m is called the slope of the line.

$(0, b)$ is the y-intercept of the line.

E.g. $f(x) = 2x + 5$. Here $m = 2$; $b = 5$

x	$f(x)$	
0	5	$\rightarrow (0, 5)$
1	7	$\rightarrow (1, 7)$

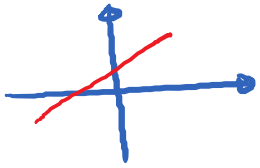
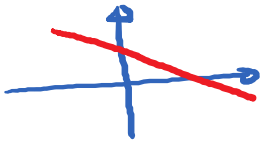


Every 1 unit change in the x-direction corresponds to 2 units change in the y-direction

Slope $m = 2$ measures the "steepness" of the line

$(0, 5)$ is the intersection of the line and the y-axis. Hence it is called the y-intercept.

To sum up, in the equation $y = mx + b$ or $f(x) = mx + b$.

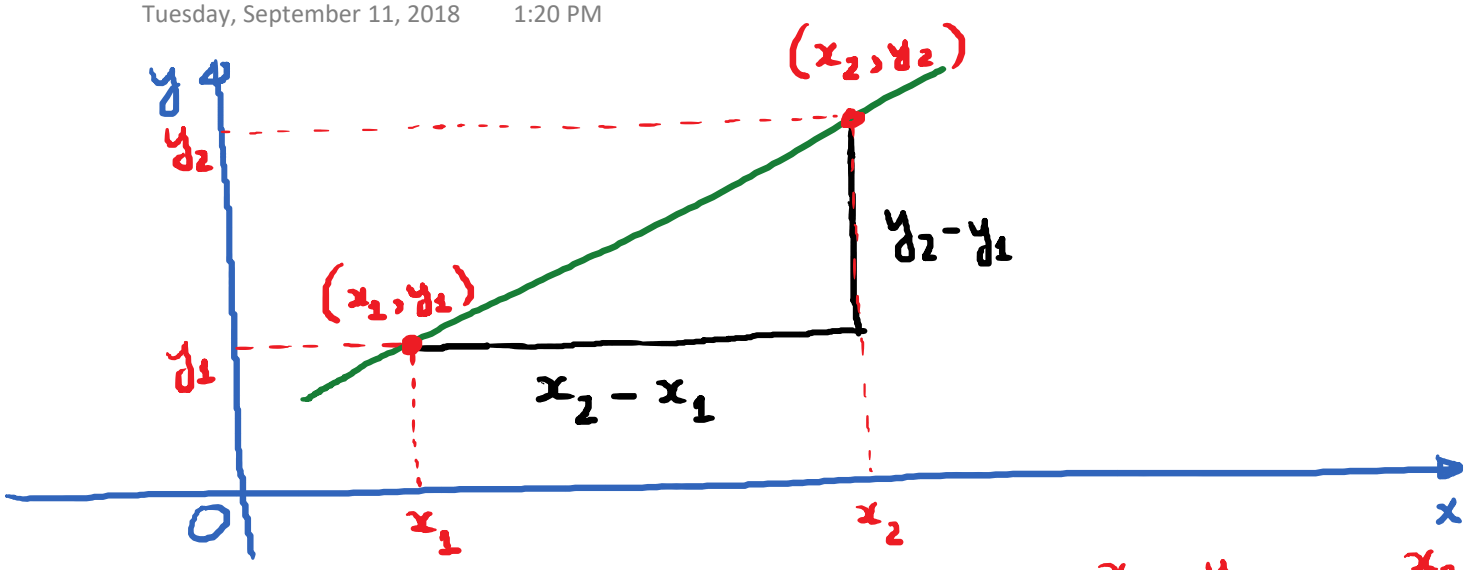
* $(0, b)$ is the y-intercept
* m = slope → measure of the steepness of the line.
  - $m > 0$  $m < 0$ 

For every 1 unit change in the x-direction, the line rise or fall by $|m|$ units

(2) Calculate the slope of a line

If (x_1, y_1) and (x_2, y_2) are points on a line L, then the slope of L is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{Rise}}{\text{Run}}$$



E.g. Find the slope of the line containing $(9, -1)$ and $(-8, -7)$ (x_1 y_1 / x_2 y_2) and interpret the result.

$$m = \frac{(-7) - (-1)}{(-8) - (9)} = \frac{-6}{-17} = \boxed{\frac{6}{17}} \approx 0.353$$

(y_2, y_1 / x_2, x_1)

Interpret: For every 1 unit change in the x-direction, the line rise by 0.353 units

Every 17-unit-change in x corresponds to 6-unit-change in y.

Ex. Find the slope of the line containing $(7,-5)$ and $(3,2)$ and interpret the result.

$$m = \frac{2-(-5)}{3-7} = \frac{7}{-4} = \boxed{-\frac{7}{4}} = -1.75$$

For every 1 unit change in the x direction, the line falls by 1.75 units

* Derive the slope-intercept form $y = mx + b$ of any linear equation

E.g. $x + 2y = 8$ → convert to slope-intercept.

Idea: Get y by itself.

$$2y = -x + 8 \rightarrow y = \frac{-x+8}{2}$$

$$\rightarrow y = \boxed{-\frac{1}{2}}x + \boxed{4}$$

→ Slope $= -\frac{1}{2}$

→ y-intercept: $(0,4)$.