2.7. Find Equations of Lines and Some Applications

Objectives : ① Find an equation of a line given:

    * Slope and y-intercept.

    * Slope and a point on the line

    * 2 points on the line

    * A point on the line and another line parallel or perpendicular to it

② Some applications

① * Find the equation of a line given the slope and y-intercept.

Q : Find an equation of the line with slope -7 and y-intercept $(0, 5)$

Answer: $y = -7x + 5$

E.g. Find an equation of the line with slope 0 and y-intercept $(0, -2018)$

Answer: $\boxed{y = -2018}$

* Find an equation of a line given
- slope
- a point on it.

E.g. Find an equation of the line with slope $\boxed{\dfrac{2}{3}}$ and passes through $(4, -9)$.

Sol: $y = mx + b$; $m = \dfrac{2}{3}$

$$y = \frac{2}{3}x + b \longrightarrow -9 = \frac{2}{3} \cdot \left(\frac{4}{1}\right) + b$$

-9 ↑ 4 ↑

$$-9 = \frac{8}{3} + b$$

$$b = -9 - \frac{8}{3} = -\frac{35}{3}.$$

$$\boxed{y = \frac{2}{3}x - \frac{35}{3}}$$

2nd way to solve this:

Using point - slope form:
$$m$$
$$(x_1, y_1)$$

① Point - Slope equation of line:

$$\boxed{y - y_1 = m(x - x_1)}$$

② Get y by itself to obtain the slope-intercept form.

In our problem: $m = \frac{2}{3}$. Point $(\underset{x_1}{4}, \underset{y_1}{-9})$

Step 1: $y - (-9) = \frac{2}{3}(x - 4)$ (Point - Slope form)

$\underbrace{}_{y_1} \quad \underbrace{\phantom{\frac{2}{3}}}_{m} \quad \underset{x_1}{4}$

Step 2: Get y by itself:

$$y + 9 = \frac{2}{3}x - \frac{8}{3}$$

$$y = \frac{2}{3}x - \frac{8}{3} - 9$$

$$\boxed{y = \frac{2}{3} - \frac{35}{3}}$$

E.x. Find the slope-intercept equation of a line with slope -1 and passes through $(-1, 8)$

Step 1: Point-Slope form:

$$y - 8 = -1 \cdot (x - (-1))$$

Step 2: Get y by itself:

$$y - 8 = -1 \cdot (x + 1)$$

$$y - 8 = -x - 1 \longrightarrow \boxed{y = -x + 7}$$

Slope-intercept equation.

* Find an equation of the line given 2 points on the line.

E.g. Find the slope-intercept equation of the line containing the points $(-5, 3)$ and $(3, -1)$.

Step 1: Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - 3}{3 - (-5)} = \frac{-4}{8} = \boxed{-\frac{1}{2}}$$

Step 2: Choose either point and plug in point-slope equation.

$$y - (-1) = -\frac{1}{2}(x - 3)$$

Step 3: Get y by itself.

$$y + 1 = -\frac{1}{2}(x - 3) \rightarrow y + 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$\rightarrow y = -\frac{1}{2}x + \frac{3}{2} - 1 \rightarrow \boxed{y = -\frac{1}{2}x + \frac{1}{2}}$$

Process of finding the slope-intercept equation of a line given 2 points (x_1, y_1) and (x_2, y_2) on it.

Step 1: Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Step 2: Choose either point and plug in the point-slope equation:

$$y - y_1 = m(x - x_1).$$

Step 3: Get y by itself & simplify to get the slope-intercept form.

Ex. Find the slope-intercept equation of the line containing $(1, 4)$ and $(-2, 7)$.

Step 1: Slope

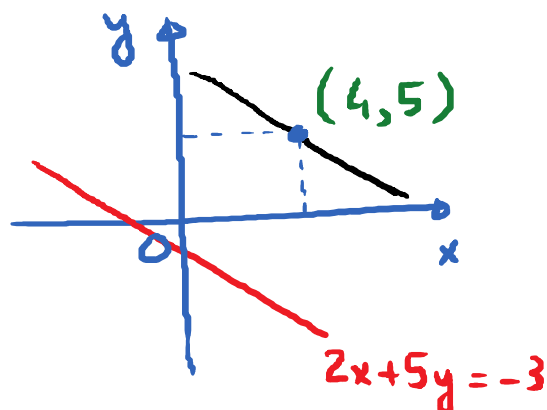$$m = \frac{7-4}{-2-1} = \frac{3}{-3} = -1$$

Step 2: Point-Slope.

$$y - 4 = -1(x - 1)$$

Step 3:

$$y - 4 = -x + 1$$

$$\boxed{y = -x + 5}$$

* Find an equation of a line give 1 point on it and another line parallel or perpendicular to it.

E.g. Find the slope-intercept equation of the line containing the point $(4,5)$ and parallel to the line $2x + 5y = -3$.

Step 1: $2x + 5y = -3$

$$5y = -2x - 3$$

$$y = -\frac{2}{5}x - \frac{3}{5}.$$

Slope $= -\frac{2}{5}$.

Step 2: Since the line we are interested in is parallel to this, its slope must be $-\frac{2}{5}$.

We are also given the point $(4,5)$ on it.

Point-Slope: $y - 5 = -\frac{2}{5}(x - 4)$

$\longrightarrow \quad y - 5 = -\frac{2}{5}x + \frac{8}{5}$

$\longrightarrow \quad y = -\frac{2}{5}x + \frac{8}{5} + 5$

$\longrightarrow \quad y = -\frac{2}{5}x + \frac{33}{5}$