

2.7. Find Equations of Lines/Linear Functions and Applications

Thursday, September 13, 2018

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Objectives: ① Find the slope-intercept equation of a line given:

- * Slope and y-intercept
- * Slope and a point on the line
- * 2 points on the line
- * A point on the line and the equation of another line parallel or perpendicular to it.

② Some applications.

① * 1st scenario. $\left. \begin{array}{l} \text{Slope} \\ \text{y-intercept} \end{array} \right\} \text{Given}$

Q: Find the slope-intercept equation of the line with slope -7 and y-intercept (0,5)

Ans:

$$y = -7x + 5$$

Q: Find an equation of the line with slope 0 and y-intercept $(0, -2018)$

Ans: $y = -2018$

* 2nd scenario $\left\{ \begin{array}{l} \text{Slope} \\ \text{A point on the line} \end{array} \right\}$ Given

E.g. Find the slope-intercept equation of the line with slope 2 and passes through $(-3, -9)$.

Sol: 1st way: $y = mx + \textcircled{b}$; $m = 2$
?

$$y = 2x + b$$

\downarrow \downarrow
 -9 -3

$$\rightarrow -9 = 2(-3) + b \rightarrow -9 = -6 + b$$

$$\rightarrow b = -3. \text{ Equation: } \boxed{y = 2x - 3}$$

2nd way to solve this:

Using the point-slope form: $\left[\begin{array}{l} m \\ (x_1, y_1) \end{array} \right]$

① Point-Slope Equation of the line with slope m and passes through (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

② Get y by itself and simplify to obtain slope-intercept.

In our problem: Given $m=2$, point $(\overbrace{-3}^{x_1}, \underbrace{-9}_{y_1})$.

Step 1: Plug in the point-slope equation:

$$y - \underbrace{(-9)}_{y_1} = \underbrace{2}_m \cdot (x - \underbrace{(-3)}_{x_1})$$

Step 2: Simplify, get y by itself.

$$y + 9 = 2(x + 3) \rightarrow y + 9 = 2x + 6$$

$$\rightarrow \boxed{y = 2x - 3} \leftarrow \text{slope-intercept.}$$

Ex. Find the slope-intercept equation of the line with slope -1 and passes through $(-1, 8)$.

Step 1: Plug in point-slope equation:

$$y - 8 = -1 \cdot (x - (-1))$$

Step 2: Get y by itself

$$y - 8 = -1 \cdot (x + 1)$$

$$\rightarrow y - 8 = -x - 1 \rightarrow \boxed{y = -x + 7}$$

* 3rd scenario: Given 2 points.

E.g. Find the slope-intercept equation of the line containing the points $(-5, 3)$ and $(3, -1)$.

Step 1: Find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{3 - (-5)} = \frac{-4}{8} = \boxed{-\frac{1}{2}}$$

Step 2: Choose either point and plug in point-slope equation: Say, I choose $(3, -1)$

$$y - (-1) = -\frac{1}{2}(x - 3)$$

Step 3: Get y by itself:

$$y + 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2} - 1$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

Process for finding slope-intercept equation given 2 points

$(x_1, y_1) ; (x_2, y_2)$

Step 1: Find Slope : $m = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Select either point and plug in the point-slope equation.

$$y - y_1 = m(x - x_1)$$

Step 3: Get y by itself and simplify to get the slope-intercept form.

E.x. Find the slope-intercept equation of the line containing $(3, 8)$ and $(-2, 7)$.

Step 1: Slope. $m = \frac{7-8}{-2-3} = \frac{-1}{-5} = \boxed{\frac{1}{5}}$

Step 2: Point-Slope: $y - 8 = \frac{1}{5}(x - 3)$

Step 3: Slope-intercept: $y - 8 = \frac{1}{5}x - \frac{3}{5}$

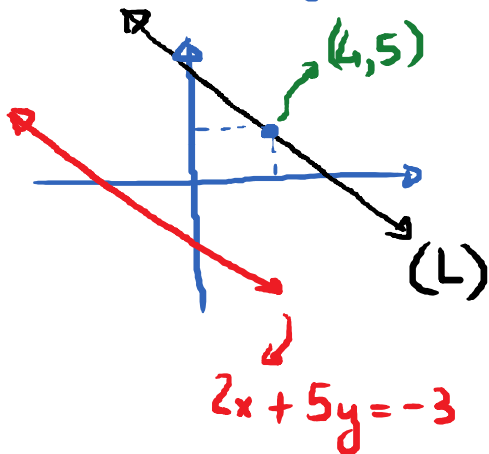
$$\rightarrow y = \frac{1}{5}x - \frac{3}{5} + \frac{8 \cdot 5}{1 \cdot 5}$$

$$\rightarrow \boxed{y = \frac{1}{5}x + \frac{37}{5}}$$

4th scenario: $\left. \begin{array}{l} \text{1 point on (L)} \\ \text{Line // or } \perp \text{ to (L)} \end{array} \right\} \text{Given}$

E.g. Find the slope-intercept equation of the line (L) containing the point (4,5) and parallel to the line

$$2x + 5y = -3.$$



Step 1: $2x + 5y = -3$

$$\rightarrow 5y = -2x - 3$$

$$\rightarrow y = -\frac{2}{5}x - \frac{3}{5}$$

$$\text{Slope} = -\frac{2}{5}.$$

Step 2: Since (L) is parallel to this, the slope of (L) must be $-\frac{2}{5}$.

Point (4,5) is on (L). So, pt-slope:

$$y - 5 = -\frac{2}{5}(x - 4)$$