7.1. Symmetry

Thursday, September 27, 2018 11:01 AM

Objectives: 1 Algebraic Tests of Symmetry.

2 Even Functions and Odd Functions

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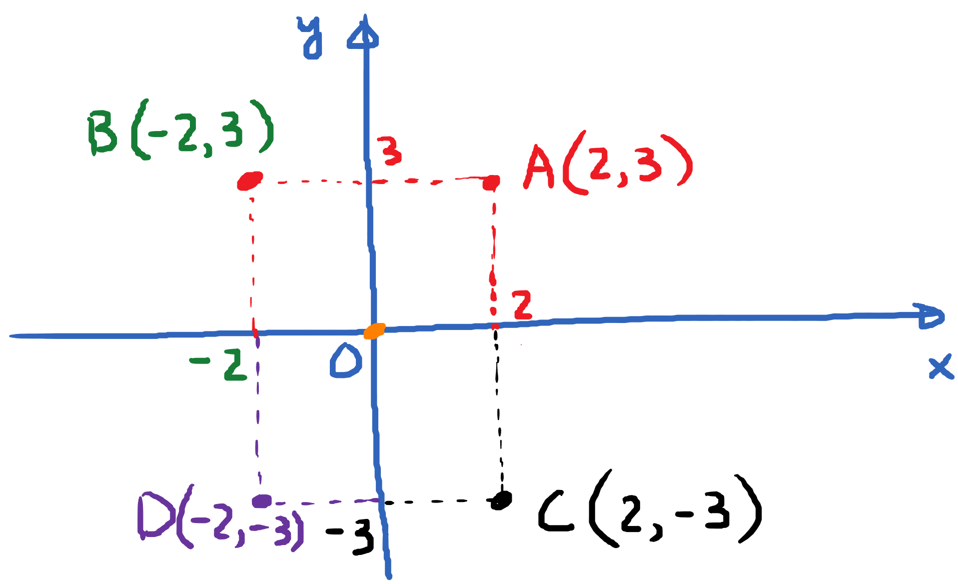


(x, y) and $(-x, y)$ are symmetric w.r.t the y-axis

(x, y) and $(x, -y)$ are symmetric w.r.t the x-axis

(x, y) and $(-x, -y)$ are symmetric w.r.t. the Origin

To test whether an equation has symmetry with respect to

1. y-axis: Replace x with −x in the equation and simplify. If we obtain the same equation, the answer is yes. Otherwise, no.
2. x-axis: Replace y with −y in the equation.
3. Origin: Replace x with −x and y with −y in the equation.

E.g. $x^2 + y^4 - 2y^2 = 8$.

Q: Test for symmetry w.r.t. x-axis, y-axis and the origin.

* x-axis: Replace y by −y:

$x^2 + (-y)^4 - 2(-y)^2 = 8$ (Replace y by -y)

$\underbrace{x^2 + y^4 - 2y^2 = 8}$ (Simplify)

→ Same as the original equation

→ Conclusion: It has symmetry w.r.t. x-axis

* y-axis: Replace x by -x

$(-x)^2 + y^4 - 2y^2 = 8$ (Replace x by -x)

$\underbrace{x^2 + y^4 - 2y^2 = 8}$ (Simplify)

→ Same as the original equation

→ Conclusion: It has symmetry w.r.t. y-axis.

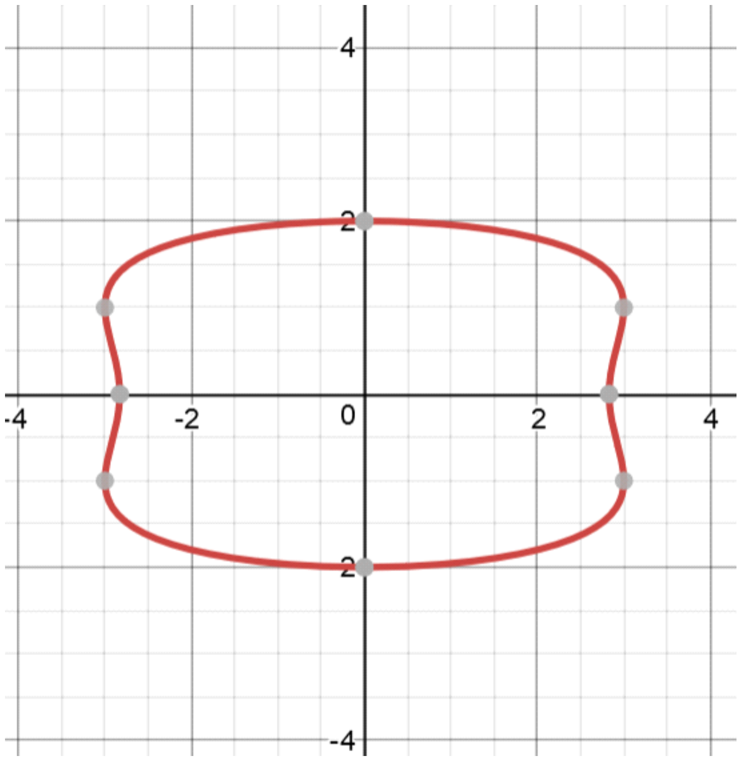
* Origin: Replace x by -x and y by -y

$(-x)^2 + (-y)^4 - 2(-y)^2 = 8$ (x by -x, y by -y)

$\underbrace{x^2 + y^4 - 2y^2 = 8}$ (Simplify)

→ same equation

→ Conclusion: It has symmetry w.r.t. origin

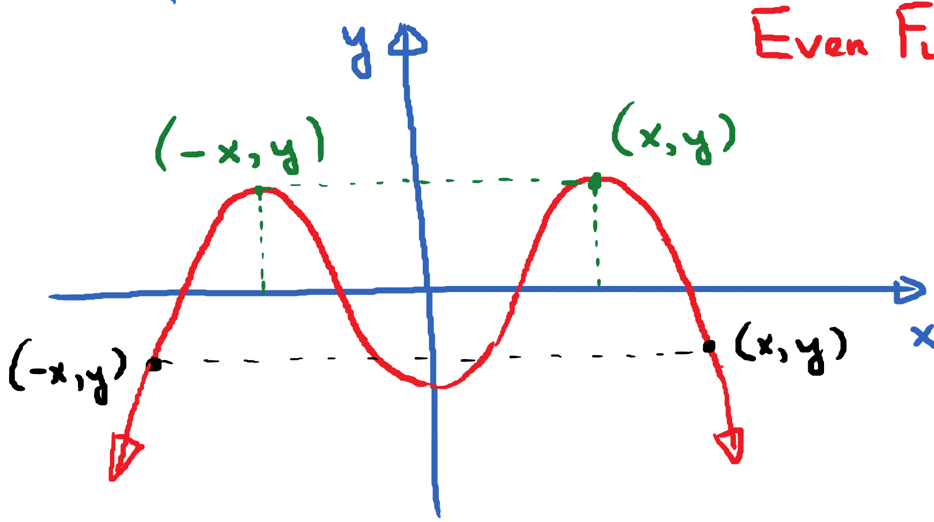


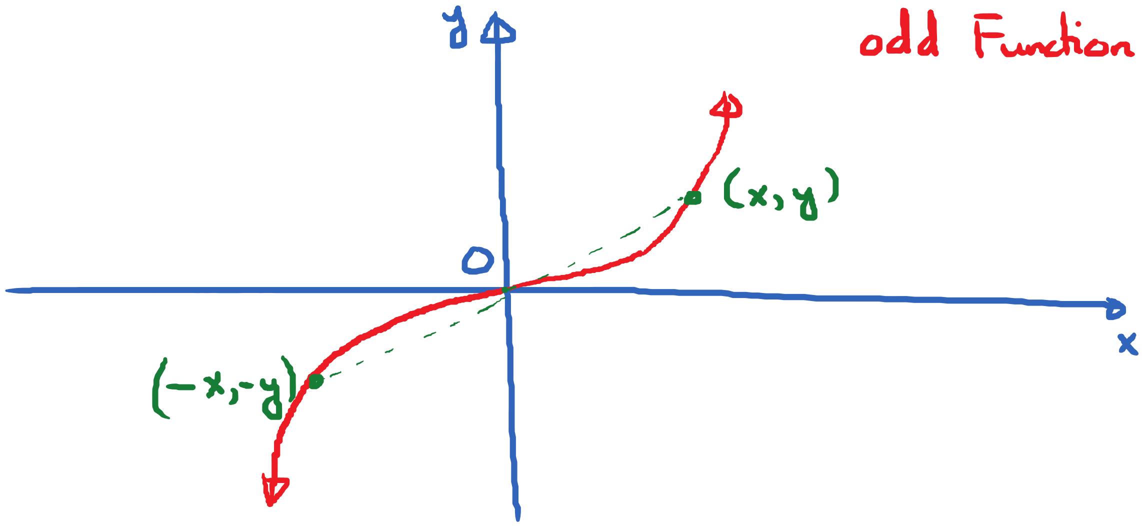
graph of

$$x^2 + y^4 - 2y^2 = 8.$$

2 Even functions and odd function

Even Function





Even : graph has symmetry w.r.t. y-axis

Odd : graph has symmetry w.r.t. origin.

Test whether a function $y = f(x)$ is even or odd

① Replace x by $-x$ to get $y = f(-x)$. If, after simplification, we get the same equation, then the function is $\boxed{\text{even.}}$

E.g. $f(x) = 5x^6 - 3x^2 - 7$. Even or odd or neither

$y = 5x^6 - 3x^2 - 7$

Replace x by -x:

$$y = 5(-x)^6 - 3(-x)^2 - 7$$

$$y = 5x^6 - 3x^2 - 7$$ → same as the original equation

→ This function is even.

Note: For polynomial functions, if all the powers of x are even, it will be even.

2 Replace x by -x and y by -y to get $-y = f(-x)$.
If, after simplification, we get the same equation, the function will be odd.

E.g. $f(x) = 5x^7 - 6x^3 - 2x$. odd or even or neither.

$$\boxed{y = 5x^7 - 6x^3 - 2x}$$ → original equation

$$-y = 5(-x)^7 - 6(-x)^3 - 2(-x)$$

$$\boxed{-y = -5x^7 + 6x^3 + 2x}$$

Multiply both sides by -1

$$\underbrace{y = 5x^7 - 6x^3 - 2x}$$

same as the original equation

<u>Note</u>: For polynomial functions, if all the powers of x are odd, then the function is odd.