7.1. Symmetry

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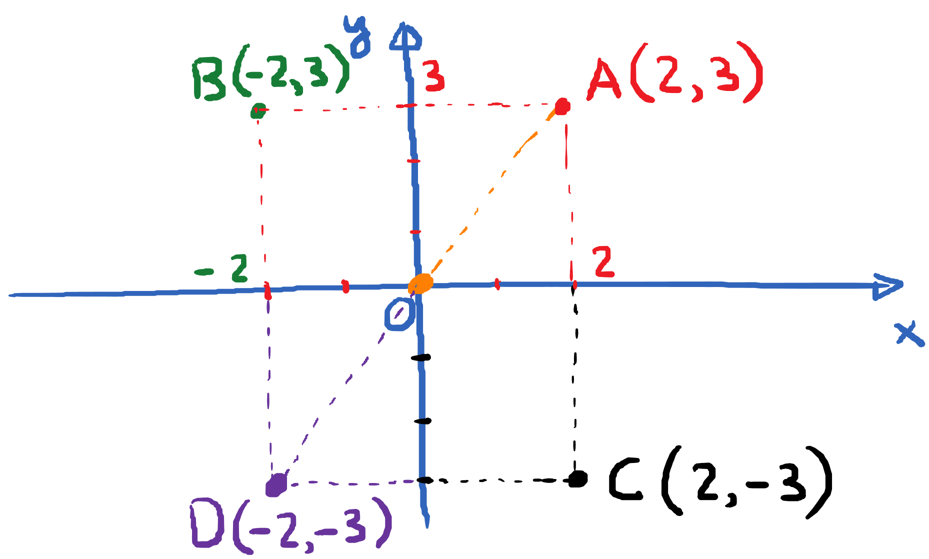
Objectives: (1) Algebraic Tests of Symmetry

(2) Even Functions and Odd Functions

(1)



(x,y) and $(-x,y)$ are symmetric with respect to y-axis

(x,y) and $(x,-y)$ are symmetric w.r.t. x-axis

(x,y) and $(-x,-y)$ are symmetric w.r.t. origin.

To test whether an equation has symmetry w.r.t.

* y-axis: Replace x by $-x$ in the equation and simplify.
If we obtain the same equation, then it has the symmetry w.r.t. y-axis. Otherwise, it does not

* x-axis : Replace y by -y in the equation and simplify.
* Origin : Replace x by -x and y by -y in the equation and simplify.

E.g. $x^2 + y^4 - 2y^2 = 8$

Q: Test for symmetry w.r.t. x-axis, y-axis and the origin

* x-axis: Replace y by -y

$x^2 + (-y)^4 - 2(-y)^2 = 8$ (Replace y by -y)

$\underbrace{x^2 + y^4 - 2y^2 = 8}$ (Simplify)

→ Same as the original equation

→ Conclusion: it has symmetry w.r.t. x-axis

* y-axis : Replace x by -x

$(-x)^2 + y^4 - 2y^2 = 8$ (Replace x by -x)

$\underbrace{x^2 + y^4 - 2y^2 = 8}$ (Simplify)

→ Same as the original equation.

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→ Conclusion: it has symmetry w.r.t. y-axis

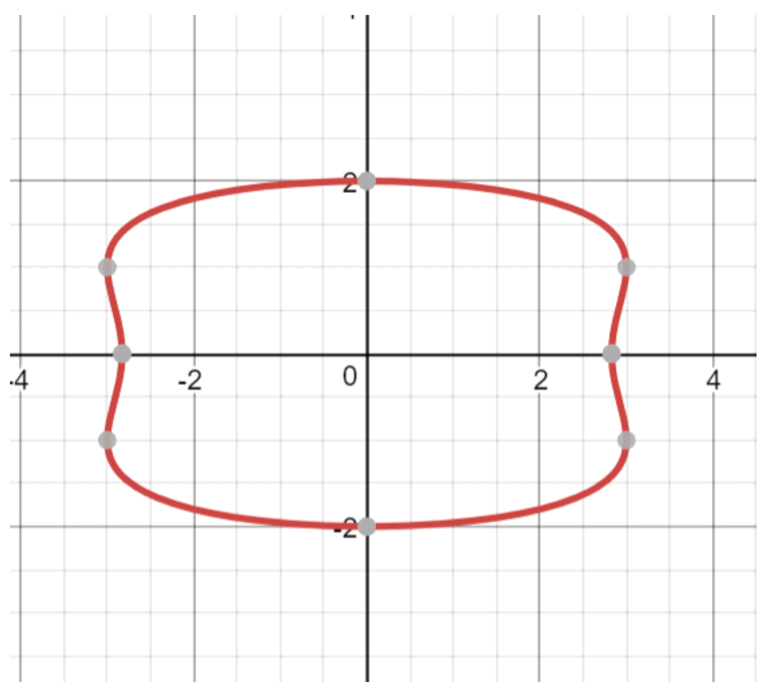
* Origin: Replace x by -x and y by -y.

$$(-x)^2 + (-y)^4 - 2(-y)^2 = 8$$

$$\underbrace{x^2 + y^4 - 2y^2 = 8}$$

→ Same as original equation

→ Conclusion: it has symmetry w.r.t. the origin

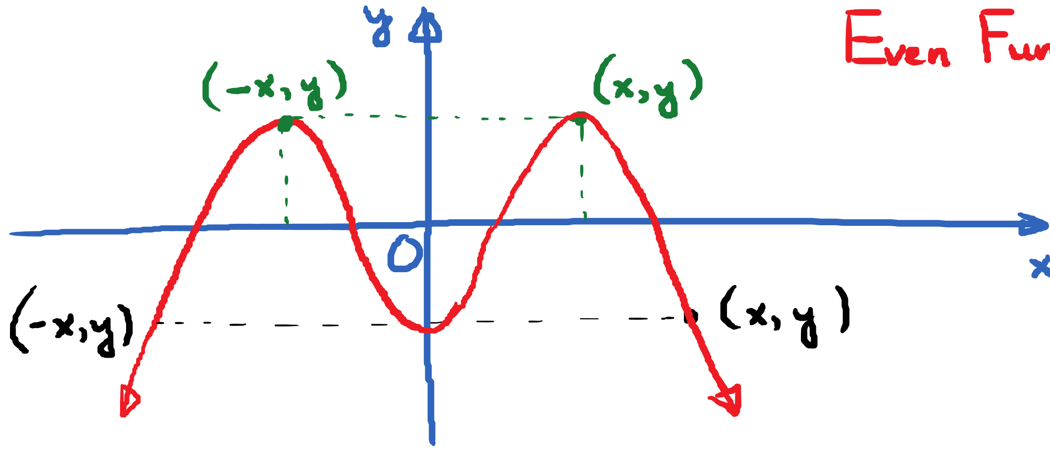


graph of

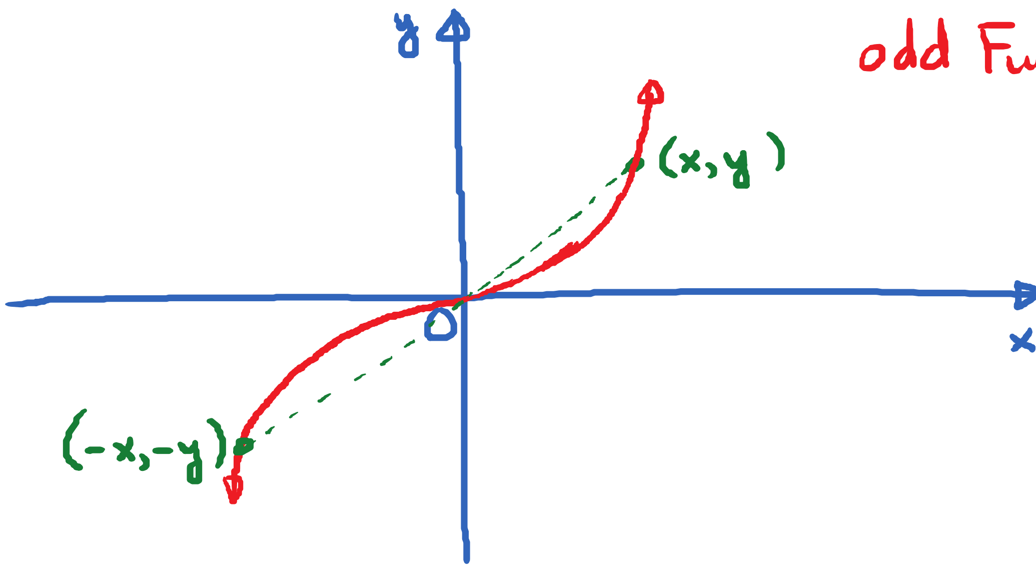
$x^2 + y^4 - 2y^2 = 8$

② Even functions and Odd Functions

Even Function



odd Function



Even Function: graph has symmetry w.r.t. y-axis

Odd Function: graph has symmetry w.r.t. origin

Given the formula $y = f(x)$, can we test algebraically whether the function is odd or even.

Replace x by $-x$ in the formula for $f(x)$

1. If $f(-x) = f(x)$, then function is even.
2. If $f(-x) = -f(x)$, then function is odd.
3. Neither 1 nor 2, the function is neither odd nor even.

E.g. $f(x) = \boxed{5x^6 - 3x^2 - 7}$. Is this function odd or even or neither?

$$f(-x) = 5(-x)^6 - 3(-x)^2 - 7$$
$$= \boxed{5x^6 - 3x^2 - 7} \longrightarrow \text{formula for } f(-x)$$

So, $f(x) = f(-x)$.

Conclusion: f is even.

Note: For polynomial functions, if all the powers of x are even, then the function is even.

E.g. $g(x) = \boxed{5x^7 - 6x^3 - 2x}$. Is g even, odd, neither?

$g(-x) = 5(-x)^7 - 6(-x)^3 - 2(-x)$ (Replace x by -x)

$= \boxed{-5x^7 + 6x^3 + 2x}$ $\rightarrow$ formula for $g(-x)$

So, $g(-x) = -g(x)$. So, g is odd.