(3) Vertical Stretching and Shrinking

For $a > 0$

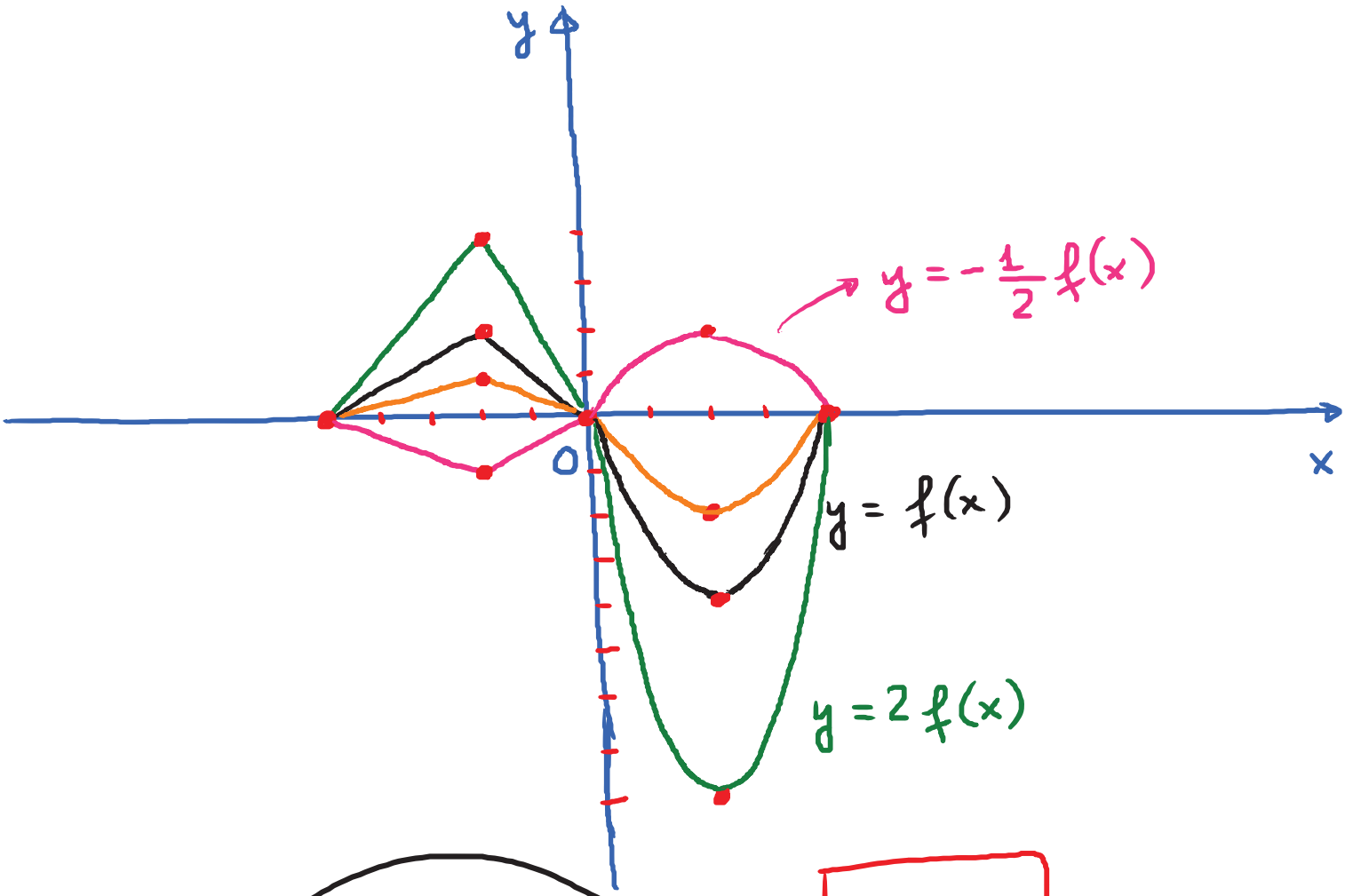
The graph of $y = a\,f(x)$ can be obtained from the graph of $y = f(x)$ by:

* Stretching vertically if $a > 1$
* Shrinking vertically if $a < 1$.

* If $a < 0$, the graph is also reflected across the x-axis

Q: Use the graph of $y = f(x)$ to obtain the graph of $y = \boxed{2} f(x)$; $y = \boxed{\frac{1}{2}} f(x)$; and $y = \boxed{-\frac{1}{2}} f(x)$

$a = 2 > 1$ $\quad a = \frac{1}{2} < 1$ $\quad a < 0$ and $|a| < 1$



Key points of f	Key points of $y = 2f(x)$	Key points of $y = \frac{1}{2} f(x)$
$(-5, 0)$	$(-5, 0)$	$(-5, 0)$
$(-2, 2)$	$(-2, 4)$	$(-2, 1)$
$(0, 0)$	$(0, 0)$	$(0, 0)$
$(2, -4)$	$(2, -8)$	$(2, -2)$
$(4, 0)$	$(4, 0)$	$(4, 0)$

* Horizontal Stretching and Shrinking

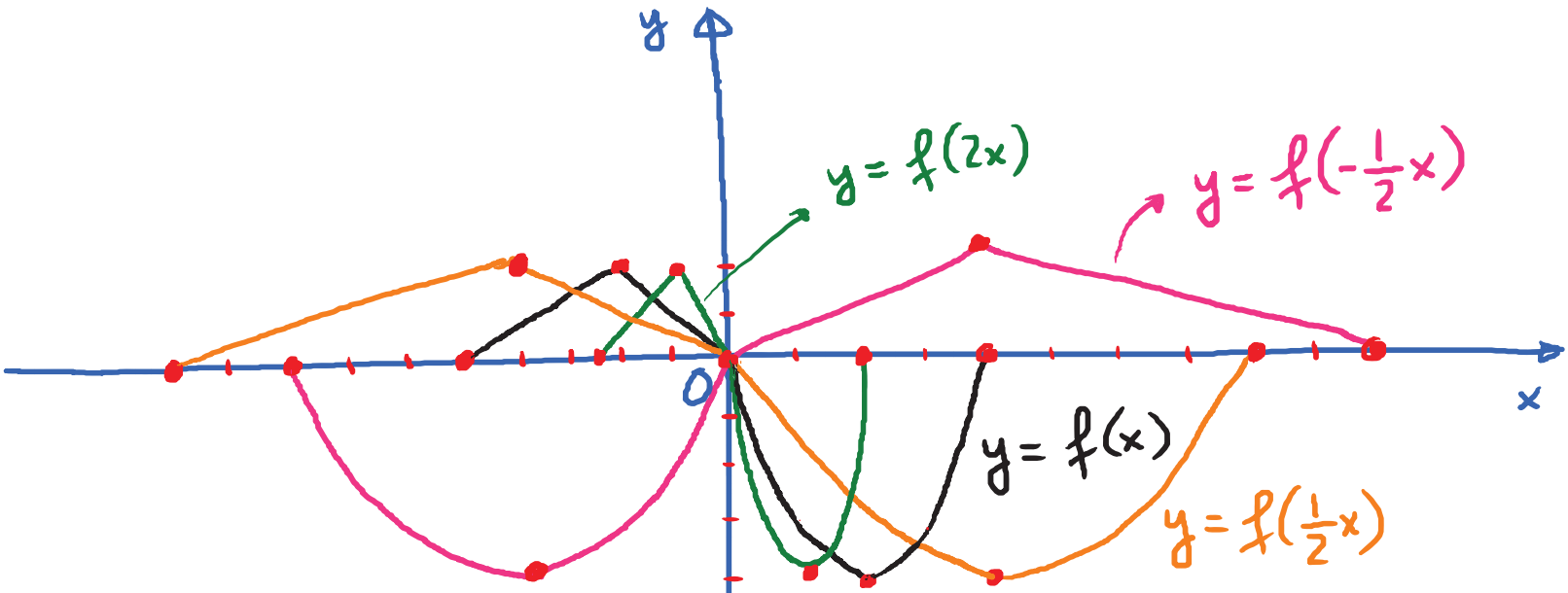
For $c > 0$

The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by:

* shrinking horizontally if $c > 1$
* stretching horizontally if $c < 1$

* If $c < 0$, the graph is also reflected across the y-axis.

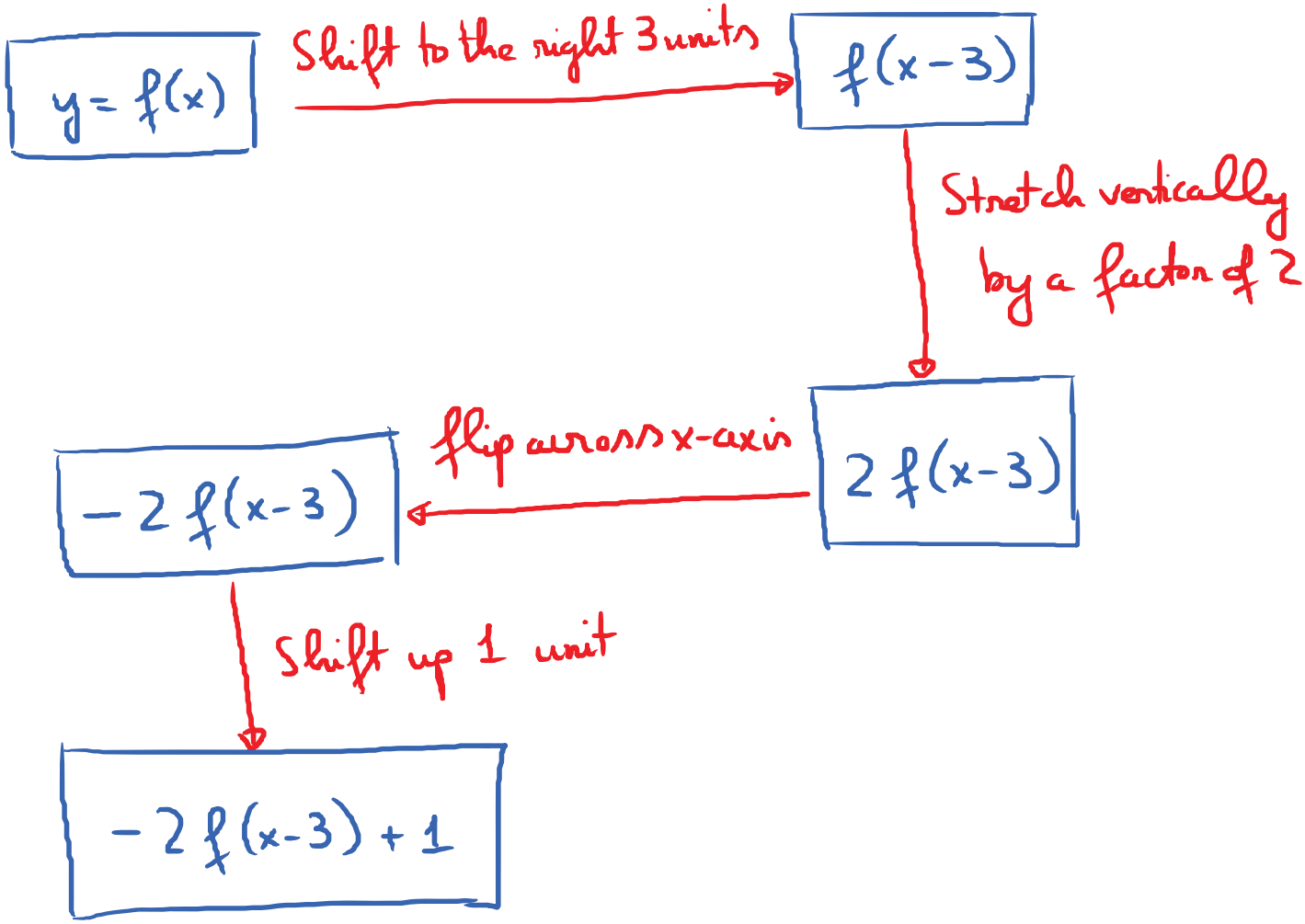
E.g. Use the graph of $y = f(x)$ to obtain the graph of $y = f(2x)$; $y = f\left(\frac{1}{2}x\right)$ and $y = f\left(-\frac{1}{2}x\right)$



Key points for $y = f(x)$	Key points for $y = f(2x)$	Key points for $y = f(\frac{1}{2}x)$
$(-5,0)$	$(-2.5,0)$	$(-10,0)$
$(-2,2)$	$(-1,2)$	$(-4,2)$
$(0,0)$	$(0,0)$	$(0,0)$
$(2,-4)$	$(1,-4)$	$(4,-4)$
$(4,0)$	$(2,0)$	$(8,0)$

E.g. $y = f(x)$

Describe how to obtain the graph of $y = -2f(x-3)+1$ from the graph of $y = f(x)$



E.g. $y = x^2$ $\longrightarrow$ $f(x) = x^2$

* Shift to the left 1 unit. $\longrightarrow$ $f(x+1) = (x+1)^2$
* Reflect across x-axis. $\longrightarrow$ $-f(x+1) = -(x+1)^2$
* Shift down 4 units. $\longrightarrow$ $-f(x+1) - 4 = -(x+1)^2 - 4$

Q: Find the formula for the resulting graph?

A: $\boxed{-(x+1)^2 - 4}$