3 * Vertical Stretching and Shrinking

For $a > 0$

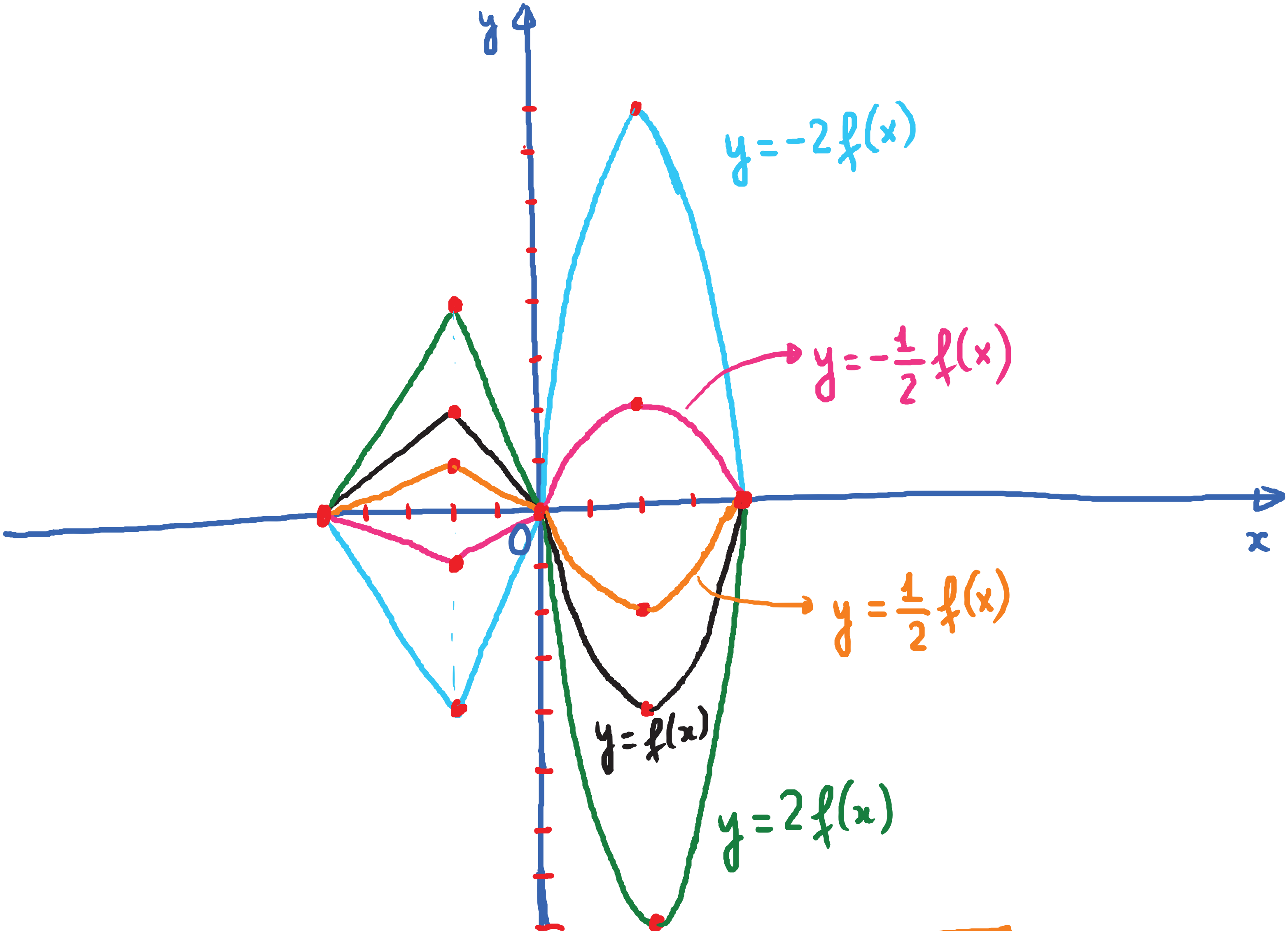
The graph of $y = f(x)$ can be obtained from the graph of $y = af(x)$ by

* Vertical Stretching if $a > 1$. (E.g. $a = 2, 3, 4, 5.7 \ldots$)
* Vertical Shrinking if $a < 1$. (E.g. $a = \frac{1}{2}, \frac{1}{3}, 0.6, \ldots$)

* Note: If $a < 0$, the graph is also reflected across the x-axis. (after stretching or shrinking)

E.g. Use the graph of $y = f(x)$ to obtain the graph of

$y = 2f(x)$; $y = \frac{1}{2}f(x)$; $y = -2f(x)$; $y = -\frac{1}{2}f(x)$



Key points of f	Key points of $y = 2f(x)$	Key points of $y = \frac{1}{2}f(x)$
$(-5,0)$	$(-5,0)$	$(-5,0)$
$(-2,2)$	$(-2,4)$	$(-2,1)$
$(0,0)$	$(0,0)$	$(0,0)$
$(2,-4)$	$(2,-8)$	$(2,-2)$
$(4,0)$	$(4,0)$	$(4,0)$

* Horizontal Stretching and Shrinking

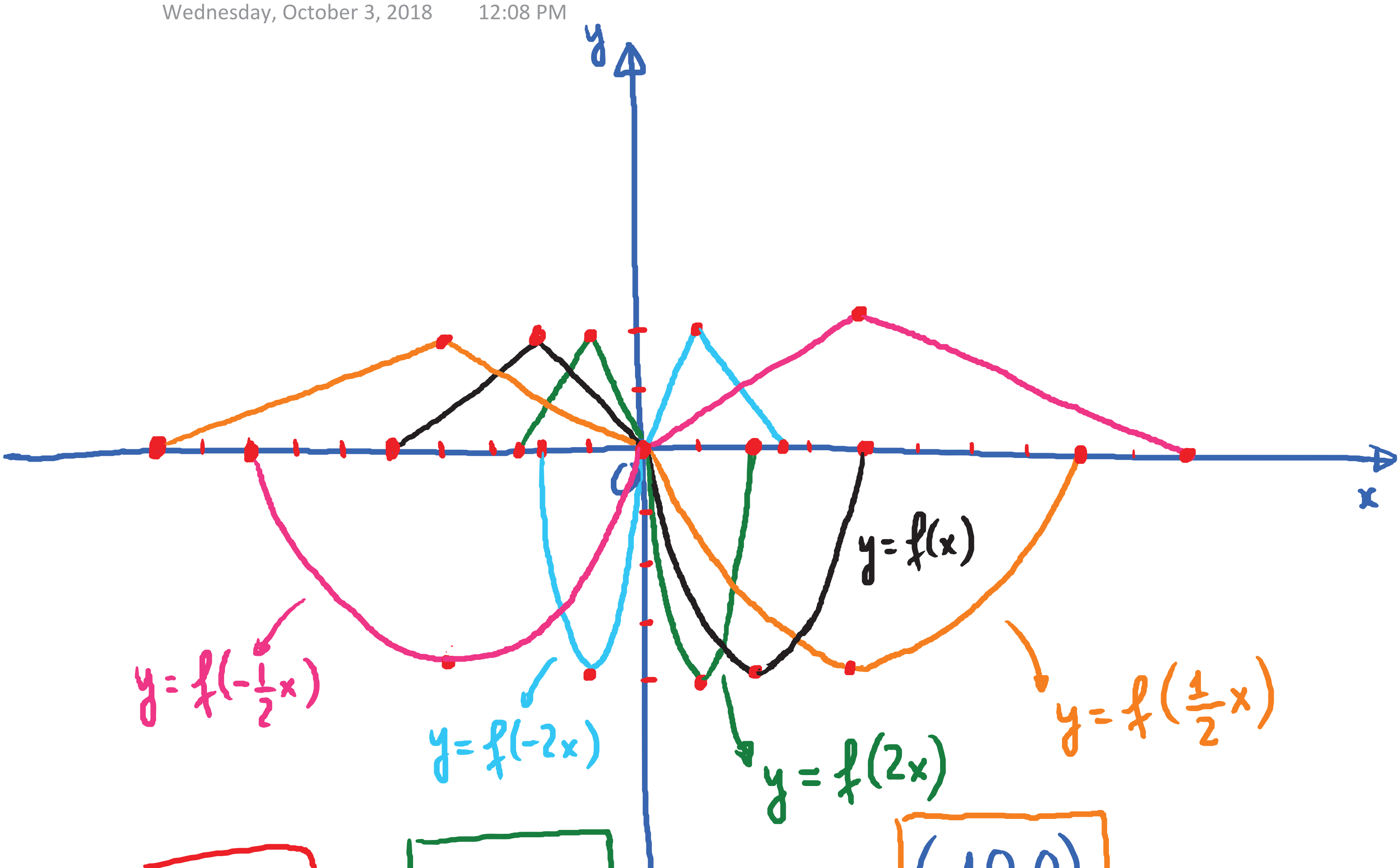
For $c > 0$

The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by:

* Horizontal shrinking if $c > 1$. (E.g. $c = 2, 3, 4.6, \ldots$)
* Horizontal stretching if $c < 1$. (E.g. $c = \frac{1}{2}, \frac{1}{3}, 0.7 \ldots$)

* **Note:** If $c < 0$, the graph is also reflected across the y-axis. (after stretching or shrinking).

E.g. Use the graph of $y = f(x)$ to obtain the graph of $y = f(2x)$; $y = f(\frac{1}{2}x)$; $y = f(-2x)$; $y = f(-\frac{1}{2}x)$



Key points of f	Key points of $y=f(2x)$	Key points of $y=f(\frac{1}{2}x)$
$(-5,0)$	$(-2.5,0)$	$(-10,0)$
$(-2,2)$	$(-1,2)$	$(-4,2)$
$(0,0)$	$(0,0)$	$(0,0)$
$(2,-4)$	$(1,-4)$	$(4,-4)$
$(4,0)$	$(2,0)$	$(8,0)$

E.g. Describe the sequence of transformations to obtain the graph $y = -2f(x-3)+1$ from the graph of $y = f(x)$.

$$\boxed{y = f(x)} \xrightarrow{\text{Shift 3 units to the right}} \boxed{y = f(x-3)}$$

$$\boxed{y = f(x-3)} \xrightarrow{\text{Stretch vertically by a factor of 2}} \boxed{y = 2f(x-3)}$$

$$\boxed{y = 2f(x-3)} \xrightarrow{\text{flip across x-axis}} \boxed{y = -2f(x-3)}$$

$$\boxed{y = -2f(x-3)} \xrightarrow{\text{Shift up by 1 unit}} \boxed{y = -2f(x-3)+1}$$

E.g. Given the graph of $f(x) = x^2$

1. Shift to the left 3 units $\rightarrow f(x+3) = (x+3)^2$
2. Reflect across the x-axis $\rightarrow -f(x+3) = -(x+3)^2$
3. Shift up 6 units $\rightarrow -f(x+3)+6 = -(x+3)^2+6$

Q: Find the formula for the resulting graph.

Ans: $\boxed{y = -(x+3)^2+6}$