7.4. Quadratic Functions, Equations, Zeros, and Models

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Objectives:
1. Quadratic Equations and Quadratic Functions
2. Completing the Square
3. Using quadratic formula
4. Discriminant
5. Applications.

A **quadratic equation** is an equation of the form:

$$\boxed{ax^2 + bx + c = 0}\ ; \ a \neq 0 \ ; \ b, c \text{ are real \#s}$$

E.g. $x^2 - 3x + 2 = 0$; $a = 1$; $b = -3$; $c = 2$

A **quadratic function** is a function of the form:

$$\boxed{f(x) = ax^2 + bx + c}\ ; \ a \neq 0 \ ; \ b, c \text{ are real \#s}$$

E.g. $f(x) = x^2 - 3x + 2$

The $\boxed{\text{zeros}}$ of a quadratic function $f(x) = ax^2 + bx + c$ are the solutions of the quadratic equations $\underbrace{ax^2 + bx + c}_{f(x)} = 0$

Equation - Solving Principles

Zero Product Principle

If $AB = 0$, then $A = 0$ or $B = 0$

E.g. $3x^2 - 7x = 0$

$\boxed{x}(\boxed{3x - 7}) = 0$

Zero product principle:

$x = 0$ or $3x - 7 = 0$

$x = \frac{7}{3}$

Solution set: $\{0, \frac{7}{3}\}$

E.g. $2x^2 - x = 3$

$\longrightarrow 2x^2 - x - 3 = 0$

$\longrightarrow$ Factor: $(\boxed{2x - 3})(\boxed{x + 1}) = 0$

$\longrightarrow$ Zero Product Principle:

$2x - 3 = 0$ or $x + 1 = 0$

$x = \frac{3}{2}$ or $x = -1$.

Solution set: $\left\{\frac{3}{2}, 1\right\}$

Square Root Principle

If $A^2 = k$, then $A = \sqrt{k}$ or

$A = -\sqrt{k}$

E.g. Solve $3x^2 = 17$

$$\longrightarrow x^2 = \frac{17}{3}$$

$\longrightarrow$ Square Root Principle: $x = \sqrt{\frac{17}{3}}$ or

$$x = -\sqrt{\frac{17}{3}}$$

E.g. Solve $2x^2 - 10 = 0$

$$2x^2 = 10 \longrightarrow x^2 = 5$$

$$\longrightarrow \boxed{x = \pm\sqrt{5}}$$

Method of Completing the Square.

E.g. Solve $\underbrace{x^2 - 6x} - 10 = 0$

$$\underbrace{x^2 - 6x + 9} = 10 + 9$$

$$(x-3)^2 = 19$$

$$x - 3 = \pm\sqrt{19} \longrightarrow x = 3 \pm \sqrt{19}$$

E.g. $\underbrace{x^2 + 8x} + 18 = 0$

$$\underbrace{x^2 + 8x + 16} = -18 + 16$$

$$(x+4)^2 = -2$$

$x + 4 = \sqrt{-2}$ or $x + 4 = -\sqrt{-2}$

$x + 4 = i\sqrt{2}$ or $x + 4 = -i\sqrt{2}$

$x = -4 + i\sqrt{2}$ or $x = -4 - i\sqrt{2}$.

(We can write this as $x = -4 \pm i\sqrt{2}$)

Quadratic Formula:

The solutions of $ax^2 + bx + c = 0$; $a \neq 0$ are given by the formula:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Solve the following equations:

$$x^2-6x-10=0$$

$a=1$; $b=-6$; $c=-10$

$$x=\frac{6\pm\sqrt{36-4\cdot 1\cdot(-10)}}{2}=\frac{6\pm\sqrt{76}}{2}$$

$$x=\frac{6\pm\sqrt{19\cdot 4}}{2}=\frac{6\pm 2\sqrt{19}}{2}=3\pm\sqrt{19}$$

E.g. (a) Solve $3x^2+2x=7$. (b) $x^2+5x+8=0$

(a) $3x^2+2x-7=0$; $a=3$; $b=2$; $c=-7$

$$x=\frac{-2\pm\sqrt{4-4\cdot 3\cdot(-7)}}{6}=\frac{-2\pm\sqrt{88}}{6}$$

$$x=\frac{-2\pm\sqrt{4\cdot 22}}{6}=\frac{-2\pm 2\sqrt{22}}{6}$$

$$\boxed{x=\frac{-1\pm\sqrt{22}}{3}}$$

(b) $x^2+5x+8=0$; $a=1$; $b=5$; $c=8$

$$\frac{-5 \pm \sqrt{25-4\cdot 1\cdot 8}}{2} = \frac{-5 \pm \sqrt{-7}}{2} = \frac{-5 \pm i\sqrt{7}}{2}$$

Discriminant.

The quantity b^2-4ac is called the discriminant of the equation $ax^2+bx+c=0$; $a \neq 0$.

$b^2-4ac > 0 \longrightarrow$ The equation has 2 real solutions.

$b^2-4ac < 0 \longrightarrow$ The equation has 2 non-real solutions

$b^2-4ac = 0 \longrightarrow$ The equation has one real solution : $x = \frac{-b}{2a}$

Application:

$$f(x) = \boxed{22.1x^2 - 72.2x + 371.9}$$

This function is used to estimate # of sales of new homes, in thousands, in the U.S., where x is the # of years after 2009.

Q: In what year were the # of sales of new homes about 563 400 or 563.4 thousands.

Set $22.1x^2 - 72.2x + 371.9 = 563.4$

→ Solve for x.

$$\overset{a}{\boxed{22.1}}x^2 \overset{b}{-72.2}x \overset{c}{-191.5} = 0$$

$$x = \frac{72.2 \pm \sqrt{(-72.2)^2 - 4\cdot(22.1)\cdot(-191.5)}}{2\cdot(22.1)}$$

$$x = \frac{72.2 + 148.8}{44.2} \quad ; \quad x = \frac{72.2 - 148.8}{44.2}$$

$$x = 5 \quad ; \quad \cancel{x = -1.733}$$

$\longrightarrow$ Year when sales = 563.4 thousands = $\boxed{2014}$