# 7.4. Quadratic Equations, Functions, Zeros and Models

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**Objectives:**

1. Quadratic Equations and Functions.
2. Completing the Square.
3. Using Quadratic Formulas.
4. Discriminant.
5. Applications.

(1) A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0; \quad a \neq 0, b, c \text{ are real \#s}$$

E.g. $x^2 - 3x + 2 = 0$ ; $a = 1$; $b = -3$; $c = 2$

A **quadratic function** is a function of the form:

$$f(x) = ax^2 + bx + c \; ; \quad a \neq 0, b, c \text{ are real \#s}$$

E.g. $f(x) = x^2 - 3x + 2$

The **zeros** of a quadratic function $f(x) = ax^2 + bx + c$ are the solutions of the equation $\underbrace{ax^2 + bx + c}_{f(x)} = 0$.

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## Equation-Solving Principles

### Zero-Product Principle

If $A \cdot B = 0$, then either $A = 0$ or $B = 0$

Apply the zero-product principle and factoring to solve quadratic equations:

E.g. $3x^2 - 7x = 0$

$x(3x - 7) = 0$

→ Zero Product Principle: $x = 0$ or $3x - 7 = 0$

$x = \frac{7}{3}$

Solution Set: $\{0, \frac{7}{3}\}$

E.g. Solve $x^2 - 3x + 2 = 0$

$$(x-1)(x-2) = 0$$

Zero product Principle

$$x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 1 \quad \text{or} \quad x = 2.$$

Solution set: $\{1, 2\}$

E.g. Find the zeros of the function $-6 = 2 \cdot (-3)$

$$f(x) = 2x^2 - x - 3$$

Sol: $f(x) = 0 \longrightarrow 2x^2 - x - 3 = 0$

$$\rightarrow 2x^2 + 2x - 3x - 3 = 0$$

$$\rightarrow 2x(x+1) - 3(x+1) = 0$$

$$\rightarrow (2x-3)(x+1) = 0$$

$2x - 3 = 0 \rightarrow x = \frac{3}{2}$ ; $x + 1 = 0 \rightarrow x = -1$

Solution set: $\left\{\frac{3}{2}, -1\right\}$

## Square Root Principle

If $A^2 = k$, then $A = \sqrt{k}$ or $A = -\sqrt{k}$

**E.g.** Solve $3x^2 - 10 = 0$

$\rightarrow 3x^2 = 10 \rightarrow x^2 = \frac{10}{3}$

$\rightarrow \boxed{x = \pm\sqrt{\frac{10}{3}}}$

**E.g.** Solve $(2x + 5)^2 = 17$

$\rightarrow 2x + 5 = \pm\sqrt{17}$

$\rightarrow 2x = -5 \pm \sqrt{17} \rightarrow \boxed{x = \frac{-5 \pm \sqrt{17}}{2}}$

# Method of Completing the Square

**E.g.** Solve $x^2 - 6x - 10 = 0$

$$x^2 - 6x + 9 = 10 + 9$$

$$(x-3)^2 = 19$$

$$\rightarrow x - 3 = \pm\sqrt{19} \rightarrow \boxed{x = 3 \pm \sqrt{19}}$$

**E.g.** $x^2 + 8x + 18 = 0$

$$x^2 + 8x + 16 = -18 + 16$$

$$(x+4)^2 = -2$$

$$x + 4 = \pm\sqrt{-2}$$

$$x + 4 = \pm i\sqrt{2} \rightarrow \boxed{x = -4 \pm i\sqrt{2}}$$

non-real solutions

# Quadratic Formula

The solutions of $ax^2+bx+c=0$; $a\neq 0$ are given by the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

E.g. Solve the given equation and completely simplify the solution:

(a) $3x^2+2x=7$ (b) $x^2+5x+8=0$

Sol: (a) $3x^2+2x-7=0$; $a=3$; $b=2$; $c=-7$

$$x=\frac{-2\pm\sqrt{4-4\cdot 3\cdot(-7)}}{6}=\frac{-2\pm\sqrt{88}}{6}=\frac{-2\pm\sqrt{4\cdot 22}}{6}$$

$$x=\frac{-2\pm 2\sqrt{22}}{6}=\boxed{\frac{-1\pm\sqrt{22}}{3}}$$

(b) $x^2+5x+8=0$ ; $a=1$ ; $b=5$ ; $c=8$

$$x=\frac{-5\pm\sqrt{25-4\cdot1\cdot8}}{2}=\frac{-5\pm\sqrt{-7}}{2}$$

$$\boxed{x=\frac{-5\pm i\sqrt{7}}{2}}$$

## Discriminant

The quantity $b^2-4ac$ is called the discriminant of the equation $ax^2+bx+c=0$.

$\boxed{b^2-4ac>0}$ → The equation has 2 real solutions

$\boxed{b^2-4ac<0}$ → The equation has 2 non-real solutions

$\boxed{b^2-4ac=0}$ → The equation has 1 real solution

$$x=-\frac{b}{2a}$$

## Application:

$f(x) = 22.1x^2 - 72.2x + 371.9$ in thousands

This function is used to estimate the # of sales of new homes in the U.S. where x is the # of years after 2009.

Q: In what year were the # of sales of new homes about 563 400 or 563.4 thousands.

Set $22.1x^2 - 72.2x + 371.9 = 563.4$ and solve for x.

$$22.1x^2 - 72.2x - 191.5 = 0$$

(a = 22.1, b = −72.2, c = −191.5)

$$x = \frac{72.2 \pm \sqrt{(-72.2)^2 - 4 \cdot (22.1) \cdot (191.5)}}{44.2}$$

$$x = \frac{72.2 + 148.8}{44.2} \quad ; \quad x = \frac{72.2 - 148.8}{44.2}$$

$x = 5$ ; ~~$x = -1.733\ldots$~~

→ corresponds to the year **2014**.