

Quadratic Formula

The solutions of $ax^2 + bx + c = 0$; $a \neq 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E.g. Solve the given equation and simplify the answer.

(a) $3x^2 + 2x = 7$

(b) $x^2 + 5x + 8 = 0$

Sol: (a) $3x^2 + 2x - 7 = 0$; $a=3$; $b=2$; $c=-7$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-7)}}{6} = \frac{-2 \pm \sqrt{88}}{6} = \frac{-2 \pm \sqrt{4 \cdot 22}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{22}}{6} = \frac{-1 \pm \sqrt{22}}{3}$$

$$\textcircled{b} \quad x^2 + 5x + 8 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 32}}{2} = \frac{-5 \pm \sqrt{-7}}{2} = \boxed{\frac{-5 \pm i\sqrt{7}}{2}}$$

Discriminant

The quantity $b^2 - 4ac$ is called the discriminant of the equation

$$ax^2 + bx + c = 0$$

① $b^2 - 4ac > 0$ → The equation has 2 real solutions.

② $b^2 - 4ac < 0$ → The equation has 2 NON-real solutions

③ $b^2 - 4ac = 0$ → The equation has 1 real solution.

$$x = -\frac{b}{2a}.$$

An Application:

$$f(x) = 22.1x^2 - 72.2x + 371.9$$

This function is used to estimate the # of sales (in thousands) of new homes in the U.S. where x is the # of years after the year 2009.

Q: In what year were the # of sales of new homes about 563 400 or 563.4 thousands.

Set $22.1x^2 - 72.2x + 371.9 = 563.4$. Then solve for

$$x. \quad \xrightarrow{\quad} \overset{a}{\boxed{22.1}}x^2 - \overset{b}{\boxed{72.2}}x - \overset{c}{\boxed{191.5}} = 0$$

$$x = \frac{72.2 \pm \sqrt{(-72.2)^2 - 4 \cdot (22.1) \cdot (-191.5)}}{44.2}$$

$$x = 5 \quad \text{on} \quad x = -1.733 \dots$$

→ corresponds to the year: 2014