7.4. Quadratic Equations, Functions, Zeros and Models

Monday, October 8, 2018 11:07 AM

Objectives: (1) Quadratic Equations and Functions.

(2) Completing the Square.

(3) Using Quadratic Formula.

(4) Discriminant.

(5) Application.

(1) A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0$$

Here a, b, c are real numbers and $a \neq 0$.

E.g. $x^2 - 3x + 2 = 0$. $a = 1$; $b = -3$; $c = 2$

A **quadratic function** is a function of the form:

$f(x) = ax^2 + bx + c$; a, b, c are real #s and $a \neq 0$

E.g. $f(x) = x^2 - 3x + 2$.

The **zeros** of a quadratic function $f(x) = ax^2 + bx + c$ are the solutions of the equation $\underbrace{ax^2 + bx + c}_{f(x)} = 0$

Equation-Solving Principles

Zero-Product Principle

If $A \cdot B = 0$, then either $A = 0$ or $B = 0$

Apply the zero-product principle and factoring to solve quadratic equations.

E.g. $3x^2 - 7x = 0$. Solve this equation by factoring.

$$x(3x - 7) = 0$$

→ Zero-Product Principle

$x = 0$ or $3x - 7 = 0$

$x = \frac{7}{3}$.

Solution Set: $\{0, \frac{7}{3}\}$

E.g. Solve $x^2 - 3x + 2 = 0$ by factoring.

$$(x-1)(x-2) = 0$$

⟶ Zero-Product Principle:

$$x - 1 = 0 \text{ or } x - 2 = 0$$

$$x = 1 \text{ or } x = 2$$

Solution Set: $\{1, 2\}$

E.g. Find the zeros of the function:

$$f(x) = 2x^2 - x - 3$$

Sol: $f(x) = 0 \longrightarrow 2x^2 - x - 3 = 0.$

$$(2x-3)(x+1) = 0$$

$$\longrightarrow 2x - 3 = 0 \text{ or } x + 1 = 0$$

$$x = \frac{3}{2} \text{ or } x = -1.$$

Solution Set $\left\{\frac{3}{2}, -1\right\}$

Square Root Principle

If $A^2 = k$, then $A = \sqrt{k}$ or $A = -\sqrt{k}$

E.g. $x^2 = 4 \rightarrow x = 2$ or $x = -2$ (or $x = \pm 2$)

$x^2 = 7 \rightarrow x = \sqrt{7}$ or $x = -\sqrt{7}$ (or $x = \pm\sqrt{7}$)

E.g. Solve (a) $3x^2 - 10 = 0$

(b) $(2x+5)^2 = 17$

Sol: (a) $3x^2 - 10 \rightarrow 3x^2 = 10 \rightarrow x^2 = \frac{10}{3}$

$\rightarrow x = \pm\sqrt{\frac{10}{3}} = \pm\frac{10\sqrt{3}}{3}$

(b) $(2x+5)^2 = 17 \rightarrow 2x+5 = \sqrt{17}$ or $2x+5 = -\sqrt{17}$

$x = \frac{-5+\sqrt{17}}{2}$ or $x = \frac{-5-\sqrt{17}}{2}$

or $x = \frac{-5 \pm \sqrt{17}}{2}$

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Method of Completing the Square

E.g. Solve $x^2 - 6x - 10 = 0$.

(Take half of the coeff. of x. Square it. Add that to both sides)

$$x^2 - 6x + 9 = 10 + 9$$

$$(x-3)^2 = 19$$

$$\rightarrow x - 3 = \pm\sqrt{19} \rightarrow \boxed{x = 3 \pm \sqrt{19}}$$

E.g. $x^2 + 8x + 18 = 0$. Solve this by completing the square.

$$x^2 + 8x + 16 = -18 + 16$$

$$(x+4)^2 = -2 \rightarrow x + 4 = \pm\sqrt{-2} = \pm i\sqrt{2}$$

(imaginary unit)