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E.g. Given $f(x) = x^2 + 10x + 23$.

Q: Find the vertex. Then rewrite the equation in vertex form. Then graph the function.

By the vertex formula, $x\text{-vertex} = h = \frac{-10}{2(1)} = \boxed{-5}$

$y\text{-vertex} = k = f(-5) = (-5)^2 + 10(-5) + 23 = -2$

Vertex: $\boxed{(-5, -2)}$

$$\text{Vertex form}: \quad f(x) = \boxed{\overset{1}{a}}(x - \boxed{\overset{-5}{h}})^2 + \boxed{\overset{-2}{k}}$$

$$\boxed{f(x) = (x+5)^2 - 2.}$$

$a = 1 > 0 \longrightarrow \boxed{\text{min value} = -2}$.

Axis of symmetry: $x = -5$

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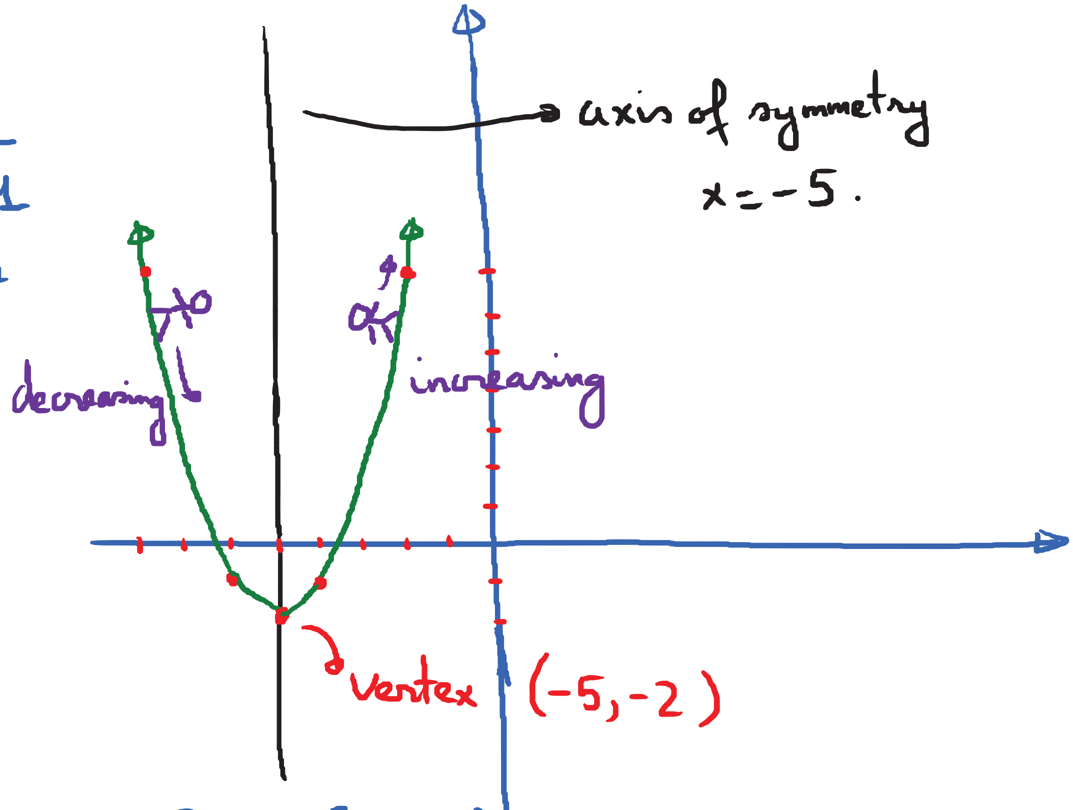
Graph:

x	y
-6	$(-6+5)^2 - 2 = -1$
-8	$(-8+5)^2 - 2 = 7$



Domain: $(-\infty, \infty)$; Range: $[-2, \infty)$

Function is increasing on $(-5, \infty)$

Function is decreasing on $(-\infty, -5)$

E.g. Given $f(x) = -x^2 + 14x - 47$.

(a) Determine whether f has a max or a min value. Then find max or min value.

(b) Find the range of f.

(c) Find the interval on which f is increasing / decreasing.

Sol: (a) Since $a = -1 < 0$, the function has a max value.

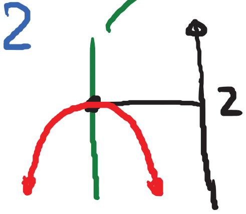
Max value $= k = y$-vertex.

$$x\text{-vertex} = -\frac{b}{2a} = \frac{-14}{2\cdot(-1)} = 7$$

$$y\text{-vertex} = f(7) = -(7)^2 + 14\cdot 7 - 47 = 2$$

So, max value of f is 2

axis of sym: $x = 7$



(b) Range $= (-\infty, 2]$

(c) Increasing interval: $(-\infty, 7)$

Decreasing interval: $(7, \infty)$

③ Applications

E.g. Height of a rocket

A model rocket is launched with an initial velocity of 100 ft/s from the top of a hill that is 20 ft high. The height, in ft, of the rocket is given by the quadratic function:

$$\boxed{s(t) = -16t^2 + 100t + 20}$$

t is time, measured in seconds.

Q: Determine the time at which the rocket reaches its maximum height and find the maximum height.

Sol: Since $a = -16 < 0$, the function will have a max.

The max value occurs at $t = -\frac{b}{2a} = \frac{-100}{2\cdot(-16)}$

So, the rocket reaches its maximum height at time

$t = 3.125$ (s)

The max height is $s(3.125) = -16(3.125)^2 + 100(3.125)$
$+20$
≈ 176.25 (ft)