7.5. Analyzing Graphs of Quadratic Functions

Tuesday, October 9, 2018 10:59 AM

Objectives: (1) Graph quadratic functions in vertex form.

(2) Vertex formula

(3) Applications

(1) Quadratic Functions in Vertex Form.

The vertex form of a quadratic function is the form

$$f(x) = a(x-h)^2 + k \qquad a \neq 0.$$

This form tells you:

* The vertex is the point (h, k)
* If $a > 0$, the parabola opens up. ∪
  If $a < 0$, the parabola opens down. ∩
* The vertical line $x = h$ is the axis of symmetry.
* If $a > 0$, the minimum value of the function is k
* If $a < 0$, the maximum value of the function is k

y-coord. of vertex ↑ k

E.g. Given $f(x) = \frac{1}{2}(x-4)^2 + 1$

Q: Find the vertex, the axis of symmetry, determine whether the function has max or min value, find the max or min value. Then graph the function.

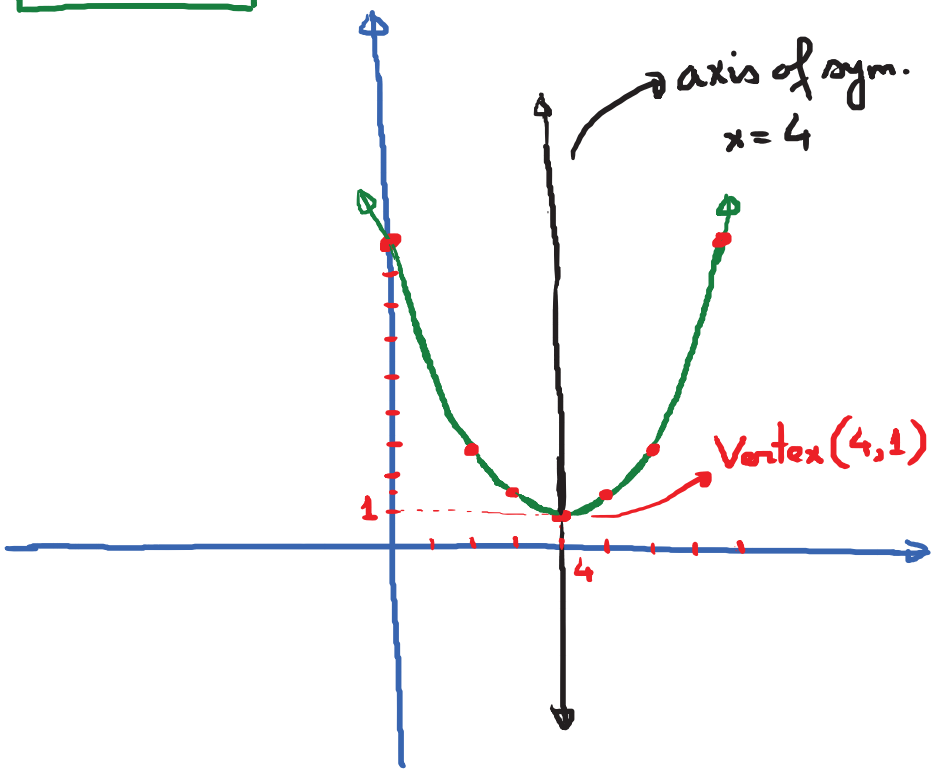
Sol: $a = \frac{1}{2}$; $h = 4$; $k = 1$

→ Vertex: $\boxed{(4,1)}$. Axis of symmetry: $\boxed{x = 4}$

$a = \frac{1}{2} > 0$ → Function has a minimum value

$\boxed{\text{Min} = 1}$

x	y
5	$\frac{1}{2}(5-4)^2 + 1 = 1.5$
6	$\frac{1}{2}(6-4)^2 + 1 = 3$
8	$\frac{1}{2}(8-4)^2 + 1 = 9$



E.g. Given $f(x) = -2\left(x + \frac{5}{2}\right)^2 + 1$.

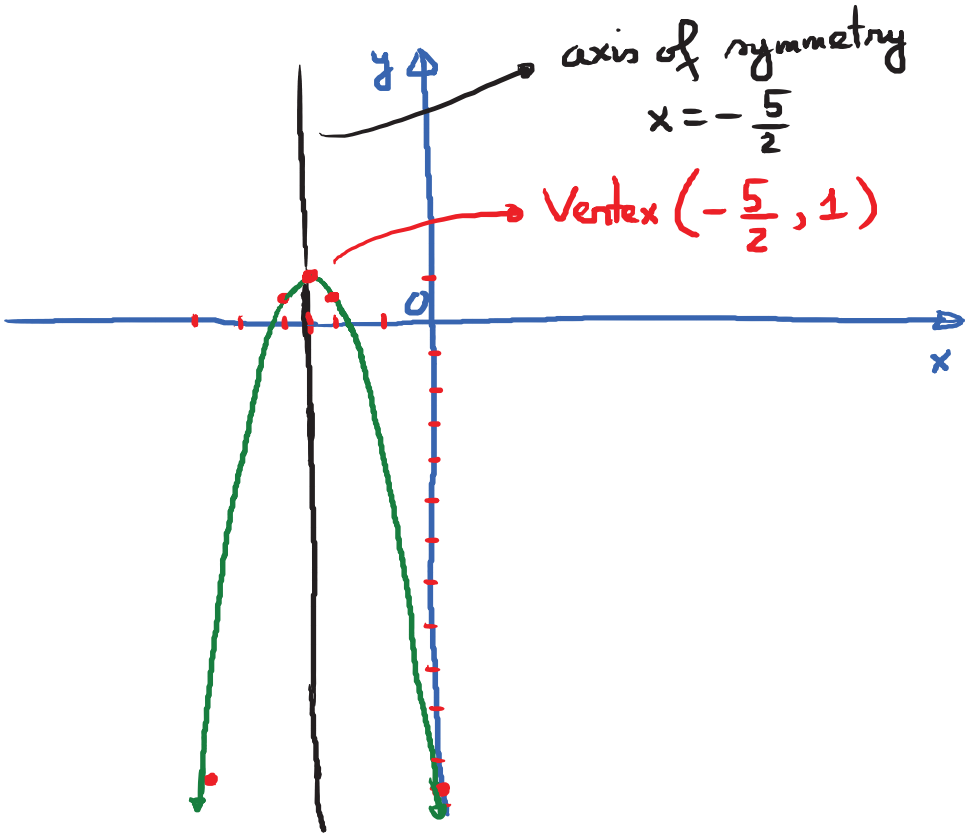
Same question as the previous example.

$a = -2$; $h = -\frac{5}{2}$ $k = 1$

Vertex: $\boxed{\left(-\frac{5}{2}, 1\right)}$ Axis of symmetry: $\boxed{x = -\frac{5}{2}}$

$a = -2 < 0 \rightarrow$ $\boxed{\text{max value of } f = 1}$

x	y
-3	$-2\left(-3+\frac{5}{2}\right)^2+1 = \frac{1}{2}$
-5	$-2\left(-5+\frac{5}{2}\right)^2+1 = -11.5$



(2) The vertex formula.

Given a quadratic function in standard form

$$f(x)=ax^2+bx+c \; ; \; a \neq 0$$

How can we turn it into the vertex form?

→ Vertex formula:

$$h = -\frac{b}{2a} \; , \quad k = f\left(-\frac{b}{2a}\right)$$

h: x-coordinate of vertex

k: y-coordinate of vertex