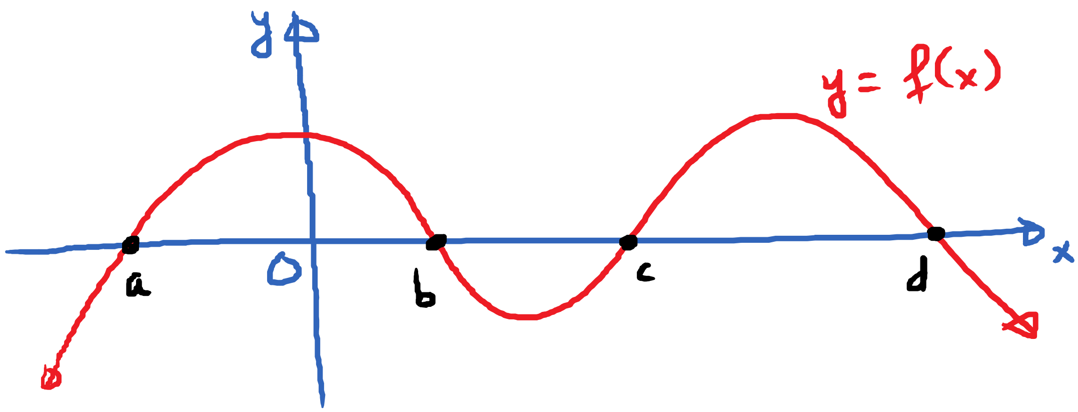
2 Zeros of Polynomial Function

We say that $x = a$ is a zero of the function $y = f(x)$ if $f(a) = 0$.



a, b, c, d are the zeros of f.

E.g. Consider the function $P(x) = x^3 + x^2 - 17x + 15$.

Q: 1 Is the # 2 a zero of P? NO

2 Is the # 0 a zero of P? NO

A: 1 $P(2) = (2)^3 + (2)^2 - 17 \cdot 2 + 15$

$= 8 + 4 - 34 + 15 = -7 \neq 0$

(2) $P(0) = 15 \neq 0$

Note: To test whether $x=a$ is a zero of $f(x)$, we plug $x=a$ into the function

- $f(a)=0 \to a$ IS a zero
- $f(a)\neq 0 \to a$ IS NOT a zero

* Finding Zeros of Polynomial Functions.

E.g. $g(x) = -(x-1)^2(x+2)^2$.

Find the zeros of g.

Key: To find the zeros of a function, we set the function equal to zero and solve for x.

Sol:

$$-(x-1)^2(x+2)^2 = 0$$

Multiply both sides by -1

$$\boxed{(x-1)^2}\boxed{(x+2)^2} = 0$$

$\longrightarrow$ Zero - Product Principle :

$(x-1)^2 = 0$ or $(x+2)^2 = 0$

$x - 1 = 0$ $\quad$ $x + 2 = 0$

$\boxed{x = 1}$ $\quad$ $\boxed{x = -2}$

Conclusion: The zeros of the function g are 1 and -2

Eg. Find the zeros of the function:

$$f(x) = (x^2 - 3x + 2)(12x - 6)^2$$

Sol: $(x^2 - 3x + 2)(12x - 6)^2 = 0$

Zero-Product Principle

$x^2 - 3x + 2 = 0$ or $(12x - 6)^2 = 0$

$(x-1)(x-2) = 0$ $\quad$ $12x - 6 = 0$

$x = 1$ or $x = 2$ $\quad$ $x = \frac{1}{2}$.

Zeros of f are $1, 2, \frac{1}{2}$.

E.g. Find the zeros of $h(x) = x^3 - 2x^2 - 9x + 18$.

Sol: $\underbrace{x^3 - 2x^2}\ \underbrace{- 9x + 18} = 0$

$x^2\boxed{(x-2)} - 9\boxed{(x-2)} = 0$ (Factoring by grouping)

$(x-2)(x^2-9) = 0$

$x - 2 = 0$ or $x^2 - 9 = 0$

$x^2 = 9 \rightarrow x = \pm 3$

$x = 2$

Zeros of h are $2, 3, -3$.

E.g. Find the zeros of $w(x) = x^4 + 4x^2 - 45$.

Sol: $x^4 + 4x^2 - 45 = 0$

$(x^2+9)(x^2-5) = 0$

$x^2 + 9 = 0$ or $x^2 - 5 = 0$

$x^2 = -9$ $\qquad x^2 = 5$

$x = \pm 3i$ $\qquad x = \pm\sqrt{5}$

w has 4 zeros:

$3i, -3i, \sqrt{5}, -\sqrt{5}$

An application:

$$\underbrace{M(t)}_{\text{amount in mg of a medication remained in the bloodstream t hours after taking the medication.}} = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t$$

Find the amount of medication in the bloodstream after

$t = 0.5\ ;\ 1\ ;\ 3;\ 5;\ 6.$

$M(0.5) = 150.2$ (mg)

$M(1) = 255$ (mg)

$M(3) = 306.9$ (mg)

$M(5) = 66$ (mg)

$M(6) = 0$ (mg)