8.1. Polynomial Functions and Models

Thursday, October 11, 2018 10:58 AM

Objectives: (1) The Leading Term Test and End Behavior

(2) Zeros of Polynomial Functions

(3) Applications

E.g. of polynomial functions:

$$f(x) = 2x^2 - 5x + 7; \quad g(x) = -\frac{1}{2}x^3 + 7x^2 - \frac{3}{5}x + 12.$$

$$h(x) = x^5 - x^2 + 17$$

E.g. of non-polynomial functions

$$f(x) = \sqrt{x} \quad ; \quad g(x) = \frac{1}{x} \quad ; \quad h(x) = \frac{x^2 + x + 1}{2x^2 - 7x + 17}$$

In general, a polynomial function is a function of the form:

$$\boxed{f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0}$$

where $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are real numbers.

a_n = leading coefficient of f.

$a_n x^n$ = leading term of f

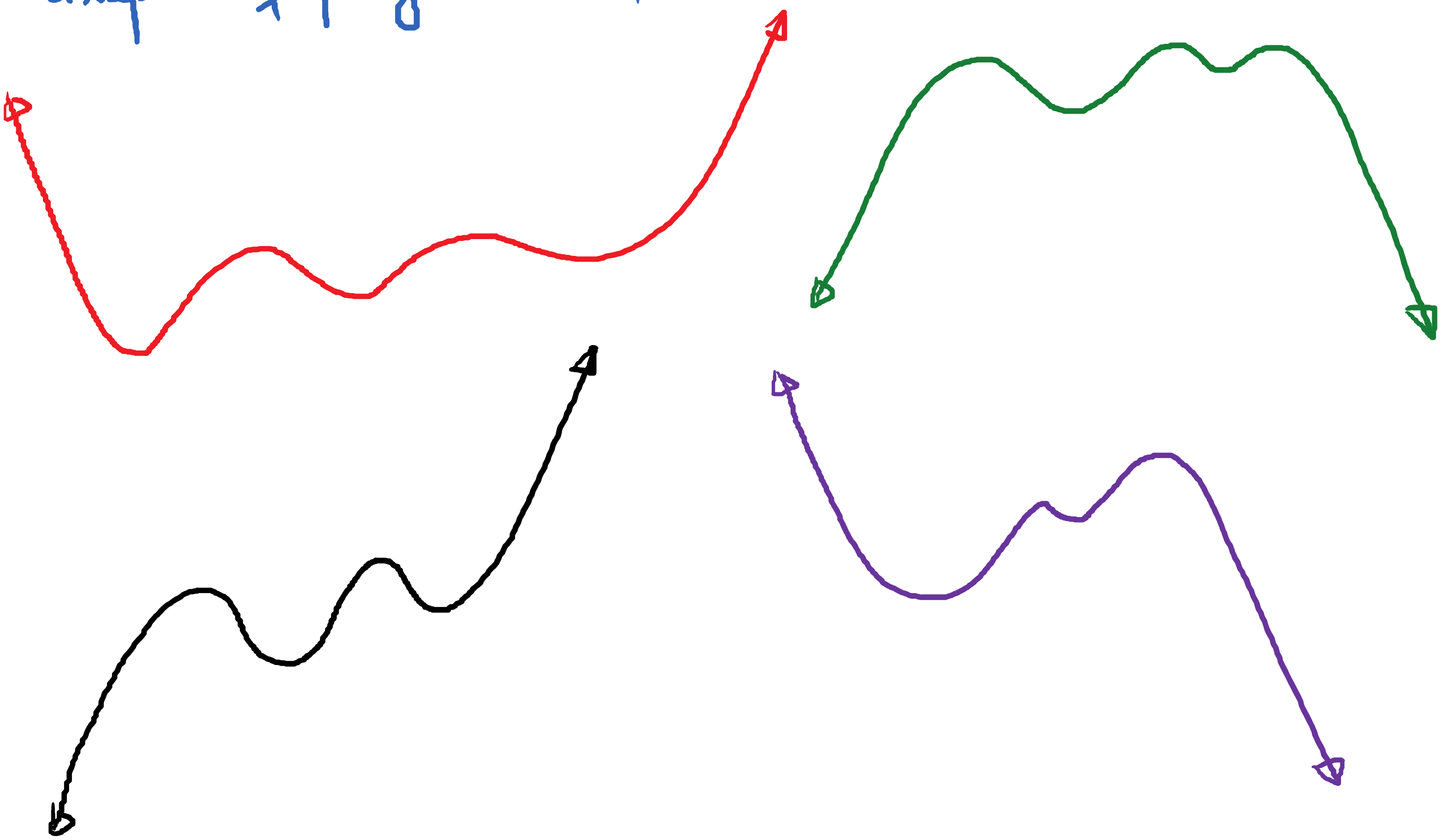
n = degree of f

a_0 = constant term of

Basic terminology.

The Leading Term Test.

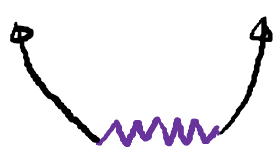
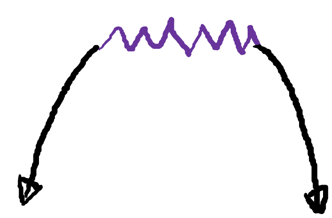
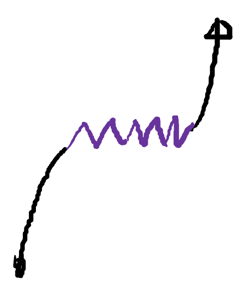
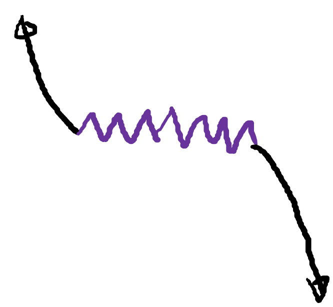
Graphs of polynomial functions:



Q: How to determine the end behavior of a polynomial function?

→ Leading Term Test.

If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ (x goes to the right on x-axis) or $x \to -\infty$ (x goes to the left on x-axis) can be described in one of the following way:

n	$a_n > 0$	$a_n < 0$
Even		
Odd		

Note: The portion of the graph is **NOT** determined by the leading term test.

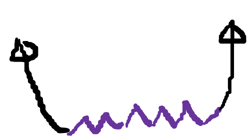
E.g. Use the leading term test to determine the end behavior of the given function.

(a) $f(x) = \boxed{3x^4} - 2x^3 + 3.$

Leading term : $3x^4$

- Leading coeff. (a_n) : $3 > 0$
- Degree (n) : 4 Even

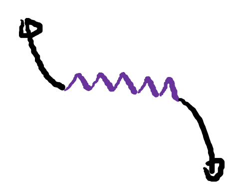
→ End behavior : rise to the left and rise to the right



(b) $g(x) = \boxed{-5}\,x^{\boxed{3}} - x^2 + 4x + 2$

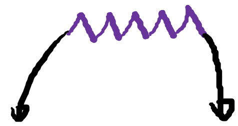
odd

$-5 < 0$



End Behavior : rise to the left, falls to the right even

(c) $g(x) = \boxed{-}x^{\boxed{6}} + \frac{1}{2}x^5 - \frac{10}{3}x^4$

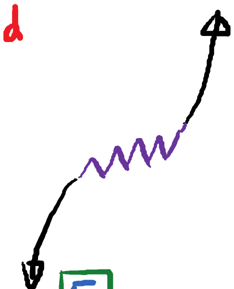


$-1 < 0$

(d) $h(x) = (\boxed{2x} - 1)(\boxed{x} + 2)(\boxed{3x} - 7)$

odd

Leading Term = $2x \cdot x \cdot 3x = \boxed{6}x^{\boxed{3}}$



> 0

(e) $j(x) = (\boxed{-x^2} + 7)\cdot(\boxed{2x} + 1)^{\boxed{3}}(\boxed{-x} + 5)^{\boxed{5}}$

Leading Term = $(-x^2)\cdot(2x)^3(-x)^5 = (-x^2)\cdot(8x^3)\cdot(-x^5)$

$= \boxed{8}x^{\boxed{10}}$ even

> 0