If we divide $f(x)$ by $x-c$, then the remainder will be equal to $f(c)$

Factor Theorem:

If we divide $f(x)$ by $x-c$ and the remainder is zero, then $x-c$ is a factor of $f(x)$.

Equivalently, if $f(c)=0$, then $x-c$ is a factor of $f(x)$.

E.g. Consider $f(x) = x^4 - 26x^2 + 25$

Q 1: Is $x-5$ a factor of $f(x)$?
(Is 5 a zero of $f(x)$)

Sol: If 5 is a zero of $f(x)$, then $f(5) = 0$. Hence, by the remainder theorem, if we divide $f(x)$ by $x-5$, we should get a remainder of 0.

→ We divide $x^4 - 26x^2 + 25$ by $x-5$ using synthetic division.

$$\begin{array}{c|ccccc} 5 & 1 & 0 & -26 & 0 & 25 \\ & & 5 & 25 & -5 & -25 \\ \hline & 1 & 5 & -1 & -5 & \boxed{0} \end{array} \rightarrow \text{Remainder}$$

Since Remainder $= 0$, 5 is a zero of $f(x)$; in other words, $x-5$ is a factor of $f(x)$.

Q2: Find the other factor of $f(x)$?

$f(x)$

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$$\underbrace{\boxed{x^4 - 26x^2 + 25}}_{\text{Dividend}} = \underbrace{\boxed{(x^3 + 5x^2 - x - 5)}}_{\text{quotient}} \cdot \underbrace{\boxed{(x-5)}}_{\text{divisor}}$$

So, the other factor of $f(x)$ is $x^3 + 5x^2 - x - 5$.

Q3: Find all the remaining zeros of $f(x)$?

Set $\underbrace{x^3 + 5x^2} \underbrace{- x - 5} = 0$

$$x^2(x+5) - (x+5) = 0$$

$$(x+5)(x^2 - 1) = 0$$

$$(x+5)(x+1)(x-1) = 0$$

$$\boxed{x = -5\ ;\ x = -1\ ;\ x = 1}$$

E.g. Consider $g(x) = x^3 - 3x^2 - 6x + 8$

Q1: Use synthetic division to determine if -2 is a zero of g.

Q2: Find the other zeros.

Q1:

$$\begin{array}{r|rrrr} & 1 & -3 & -6 & 8 \\ -2 & & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array} \longrightarrow \text{Remainder.}$$

So, -2 is a zero of g.

Q2: $x^2 - 5x + 4 = 0 \leftrightarrow (x-1)(x-4) = 0$

So, $x = 1$; $x = 4$ are the other zeros of g.