

8.3. Polynomial Division; the Remainder Theorem and the Factor Theorem

Thursday, October 5, 2018 10:58 AM

Factor Theorem

Objectives: ① Long Division

② Synthetic Division

③ Remainder Theorem and Factor Theorem.

① E.g. Use Long Division to Divide Polynomials

$$(2x^4 + 3x^3 - 7x - 10) \div (x^2 - 2x)$$

Dividend \rightarrow $2x^4 + 3x^3 - 7x - 10$ Divisor \rightarrow $x^2 - 2x$

1st term of quotient \rightarrow $\frac{2x^4}{x^2} = 2x^2$

2nd term of quotient \rightarrow $\frac{7x^3}{x^2} = 7x$

3rd term of quotient \rightarrow $\frac{14x^2}{x^2} = 14$

Remainder \rightarrow $0 + 21x - 10$

deg = 1 < 2 = deg of divisor \rightarrow Done!

Result of problem: Quotient = $2x^2 + 7x + 14$

Remainder = $21x - 10$

$$\frac{\overset{\text{Dividend}}{2x^4 + 3x^3 - 7x - 10}}{\underset{\text{Divisor}}{x^2 - 2x}} = \overset{\text{Quotient}}{2x^2 + 7x + 14} + \frac{\overset{\text{Divisor}}{21x - 10}}{x^2 - 2x}$$

Another way to write the result \rightarrow clear the denominator:
(Multiply both sides by $(x^2 - 2x)$)

$$2x^4 + 3x^3 - 7x - 10 = (2x^2 + 7x + 14)(x^2 - 2x) + 21x - 10$$

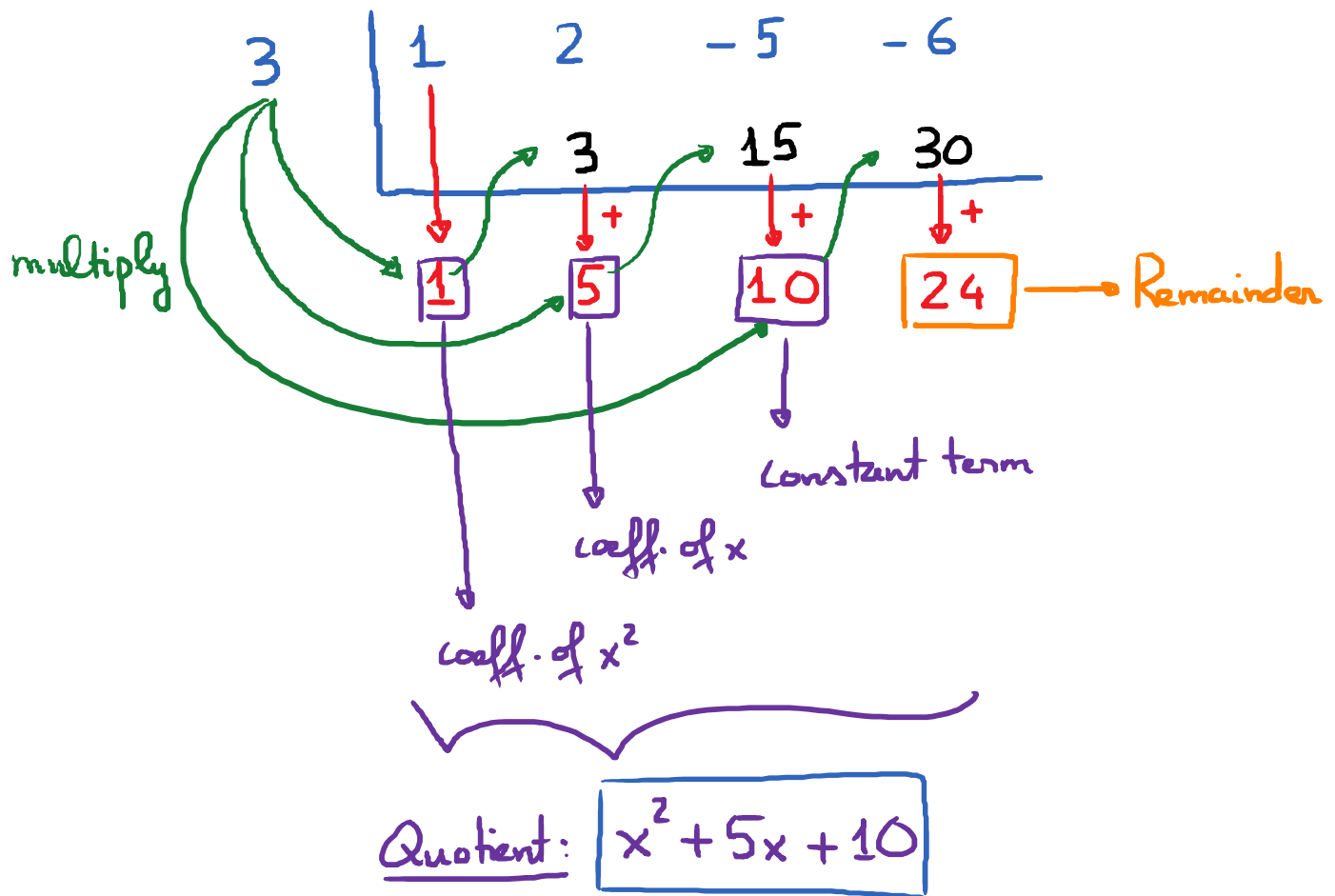
Dividend = Quotient \cdot Divisor + Remainder

② Synthetic Division.

E.g. Divide $x^3 + 2x^2 - 5x - 6$ by $x - 3$ using synthetic division

Note: We can apply synthetic division only if the divisor has the form $x - a$ or $x + a$

Sol:

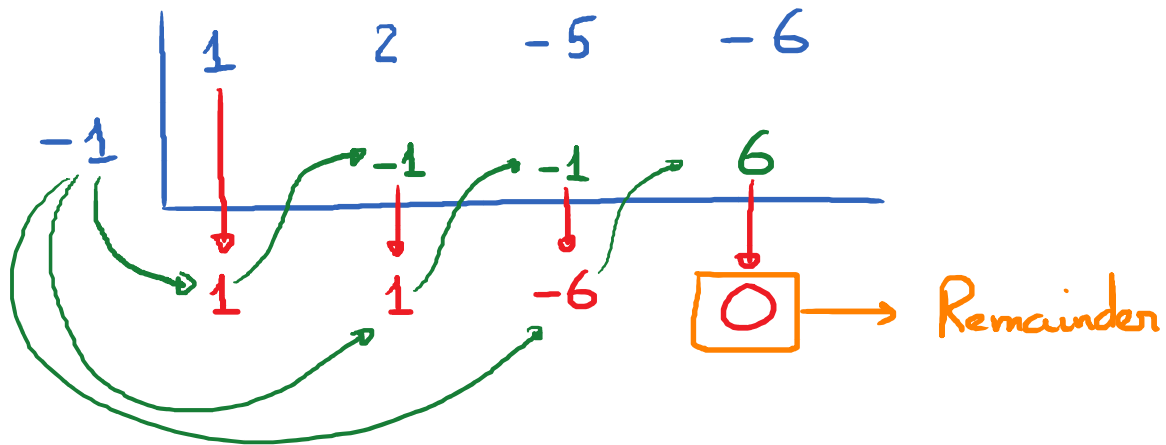


Result: $x^3 + 2x^2 - 5x + 6 = (x^2 + 5x + 10) \cdot (x - 3) + 24$

Dividend quotient divisor Remainder

E.g. Use synthetic division to divide:
 $(x^3 + 2x^2 - 5x - 6) \div (x + 1)$

Sol:



Quotient: $x^2 + x - 6$

$$\underbrace{x^3 + 2x^2 - 5x - 6}_{\text{Dividend}} = \underbrace{(x^2 + x - 6)}_{\text{quotient}} \cdot \underbrace{(x + 1)}_{\text{divisor}}$$

NO Remainder

③ Remainder Theorem and Factor Theorem

Remainder Theorem: