8.3. Polynomial Division; the Remainder Theorem and the Factor Theorem

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<u>Objectives:</u>

1. Long Division
2. Synthetic Division
3. Remainder Theorem and Factor Theorem.

1 E.g. Use Long Division to Divide Polynomials

$$\boxed{\underbrace{(2x^4+3x^3-7x-10)}_{\text{Dividend}} \div \underbrace{(x^2-2x)}_{\text{Divisor}}}$$

$$\begin{array}{r|l} & \boxed{\boxed{2x^2}+\boxed{7x}+14} \longrightarrow \text{quotient} \\ \hline \boxed{x^2}-2x & \boxed{2x^4}+3x^3 \qquad -7x-10 \\ & -(2x^4-4x^3) \\ \hline & 0+\boxed{7x^3} \qquad -7x-10 \\ & -(7x^3-14x^2) \\ \hline & 0\ \boxed{+14x^2}-7x-10 \\ & -(14x^2-28x) \\ \hline & 0+\boxed{21x-10} \longrightarrow \text{Remainder} \end{array}$$

1st term of quotient: $\dfrac{2x^4}{x^2}=\boxed{2x^2}$

2nd term of quotient: $\dfrac{7x^3}{x^2}=\boxed{7x}$

3rd term of quotient: $\dfrac{14x^2}{x^2}=\boxed{14}$

deg $=1<2=$ deg of divisor $\to$ Done!

Result of problem: Quotient $= 2x^2 + 7x + 14$

Remainder $= 21x - 10$

$$\frac{\overbrace{\boxed{2x^4 + 3x^3 - 7x - 10}}^{\text{Dividend}}}{\underbrace{\boxed{x^2 - 2x}}_{\text{Divisor}}} = \overbrace{\boxed{2x^2 + 7x + 14}}^{\text{Quotient}} + \frac{\overbrace{\boxed{21x - 10}}^{\text{Divisor}}}{x^2 - 2x}$$

Another way to write the result $\rightarrow$ clear the denominator:

(Multiply both sides by $(x^2 - 2x)$)

$$\boxed{2x^4 + 3x^3 - 7x - 10} = \boxed{(2x^2 + 7x + 14)} \cdot \boxed{(x^2 - 2x)} + \boxed{21x - 10}$$

Dividend = Quotient · Divisor + Remainder

(2) Synthetic Division.

E.g. Divide $x^3 + 2x^2 - 5x - 6$ by $x - 3$ using synthetic division

Note: We can apply synthetic division only if the divisor has the form $x - a$ or $x + a$

Sol:

$$
\begin{array}{r|rrrr}
3 & 1 & 2 & -5 & -6 \\
  &   & 3 & 15 & 30 \\
\hline
  & \boxed{1} & \boxed{5} & \boxed{10} & \boxed{24}
\end{array}
$$

multiply

Remainder

constant term

coeff. of x

coeff. of x^2

Quotient: $\boxed{x^2 + 5x + 10}$

Result: $\underbrace{x^3 + 2x^2 - 5x + 6}_{\text{Dividend}} = \underbrace{(x^2 + 5x + 10)}_{\text{quotient}} \cdot \underbrace{(x-3)}_{\text{divisor}} + \underbrace{24}_{\text{Remainder}}$

E.g. Use synthetic division to divide:

$$(x^3+2x^2-5x-6)\div(x+1)$$

Sol:

$$\begin{array}{r|rrrr} & 1 & 2 & -5 & -6 \\ -1 & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array} \longrightarrow \text{Remainder}$$

Quotient: x^2+x-6

$$\boxed{\underbrace{x^3+2x^2-5x-6}_{\text{Dividend}} = \underbrace{(x^2+x-6)}_{\text{quotient}}\cdot\underbrace{(x+1)}_{\text{divisor}}}$$

NO Remainder

(3) Remainder Theorem and Factor Theorem

Remainder Theorem: