Irrational Zeros :

If $a + c\sqrt{b}$ is a zero of f, then $a - c\sqrt{b}$ is also a zero of f

E.g. * $2 + 5\sqrt{7}$ is a zero of f. Then $2 - 5\sqrt{7}$ is also a zero of f.

* $-3 - \sqrt{11}$ is a zero of f. Then $-3 + \sqrt{11}$ is also a zero of f.

E.g. Find a polynomial function of degree 6 that has $-2+5i$, $-2i$, and $1-\sqrt{3}$ as three of its zeros.

By the property of non-real zeros:

$\boxed{-2+5i}$ is a zero $\longrightarrow$ $\boxed{-2-5i}$ is also a zero

$\boxed{-2i}$ is a zero $\longrightarrow$ $\boxed{2i}$ is also a zero

By the property of irrational zeros:

$\boxed{1-\sqrt{3}}$ is a zero $\longrightarrow$ $\boxed{1+\sqrt{3}}$ is also a zero.

Factor Theorem tells us that

$$f(x) = a_n \boxed{(x-(-2+5i))}\boxed{(x-(-2-5i))}\boxed{(x-(-2i))}\boxed{(x-2i)} \cdot \boxed{(x-(1-\sqrt{3}))}\boxed{(x-(1+\sqrt{3}))}$$

Take $a_n = 1$.

$$f(x) = \boxed{((x+2)-5i)}\boxed{((x+2)+5i)}\boxed{(x+2i)}\boxed{(x-2i)}\boxed{((x-1)+\sqrt{3})}\boxed{((x-1)-\sqrt{3})}$$

$$(A - B)(A + B) \quad (A+B)(A-B) \quad (A+B)(A-B)$$

$$= \left[(x+2)^2-(5i)^2\right] \cdot \left[x^2-(2i)^2\right] \cdot \left[(x-1)^2-(\sqrt{3})^2\right]$$

$$= \left[x^2+4x+4+25\right] \cdot \left[x^2+4\right] \cdot \left[x^2-2x+1-3\right]$$

$$= (x^2 + 4x + 29)(x^2 + 4)(x^2 - 2x - 2).$$

2 Rational Zeros Theorem

E.g. Consider $f(x) = 3x^5 + 4x^4 - 2x^3 + x^2 - 5x - 4$.

Step 1: Leading coeff. 3

Constant term. -4

Step 2: Factors of leading coeff: $\pm 1, \pm 3$

Factors of constant term: $\pm 1, \pm 2, \pm 4$

Step 3: Form a list of fractions:

$$\frac{\text{Factor of constant term}}{\text{Factor of leading coeff}}$$

$$\text{List} = \left\{ \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} \right\} = \left\{ \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3} \right\}$$

Conclusion: If f has any rational zero, it must come from the above list.

E.g. Consider $f(x) = x^3 + x^2 - 5x - 2$.

Q1: Apply the Rational Zero Theorem to make a list of all the possible rational zeros of f.

Q2: Use synthetic division to test the #s in the list to see which one (if any) is a zero of f

Q3: Find the remaining zeros of f.

Q1: leading coeff $= 1 \rightarrow$ Factors : ± 1

constant term $= -2 \rightarrow$ Factors : $\pm 1, \pm 2$.

$$\frac{\pm 1, \pm 2}{\pm 1} \rightarrow \{\pm 1, \pm 2\}$$