# 8.4. Theorems about Zeros of Polynomial Functions

Tuesday, October 30, 2018 11:13 AM

**Objective:** (1) Explain Theorems about zeros of poly. functions

(2) Rational Zero Theorem and its applications

## The factor theorem

For a polynomial $f(x)$, if $f(c) = 0$, then $x - c$ is a factor of $f(x)$

## The Fundamental Theorem of Algebra

Every polynomial of degree $n$ can be factored into a product of $n$ linear factors.

$$f(x) = a_n(x - c_1)(x - c_2)\cdots(x - c_n)$$

**E.g. 1** Find a polynomial of degree 3 having the zeros $1, 3i, -3i$.

Call the function $f(x)$

$f(1) = 0 \longrightarrow x - 1$ is a factor $f$.

$f(3i) = 0 \longrightarrow x - 3i$ is a factor $f$.

$f(-3i) = 0 \longrightarrow x + 3i$ is a factor $f$.

Since the degree is 3, these are all the factors of $f$.

So, $f(x) = \boxed{a_n}(x-1)(x-3i)(x+3i)$

leading coeff.

Problem gives no info. about leading coeff. We can take $a_n = 1$.

$f(x) = (x-1)(x-3i)(x+3i) \longrightarrow$ Expand.

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Recall: Difference between Squares Formula.

$$\boxed{(A-B)(A+B) = A^2 - B^2}$$

Back to problem:

$$f(x) = (x-1)\boxed{(\overset{A}{x} - \overset{B}{3i})(\overset{A}{x} + \overset{B}{3i})}$$

$$= (x-1)\left(x^2 - (3i)^2\right)$$

$$= (x-1)\left(x^2 - 9\underbrace{\boxed{i^2}}_{=-1}\right) = (x-1)(x^2+9)$$

$$= x^3 + 9x - x^2 - 9$$

$$\boxed{f(x) = x^3 - x^2 + 9x - 9}$$

E.g. Find a polynomial of degree 6 with

- $-1$ is a zero of multiplicity 2.
- 4 is a zero of multiplicity 1.
- 0 is a zero of multiplicity 3.

multiplicity

$(x+1)^{\boxed{2}}$ is a factor of $f$

$(x-4)$ ______

$x^3$ ______

So, $f(x) = a_n (x+1)^2 (x-4) x^3$. Take $a_n = 1$.

$$f(x) = (x+1)^2 (x-4) x^3$$

Recall: Square of a Sum.

$$\boxed{(A+B)^2 = A^2 + 2AB + B^2}$$

$$f(x) = (x^2 + 2x + 1)(x - 4)x^3$$

$$= (x^3 - 4x^2 + 2x^2 - 8x + x - 4)x^3$$

$$= (x^3 - 2x^2 - 7x - 4)x^3$$

$$= \boxed{x^6 - 2x^5 - 7x^4 - 4x^3}$$

## Properties of non-real zeros and irrational zeros

### Nonreal Zeros:

> If $a+bi$ is a zero of $f$, then its conjugate $a-bi$ is also a zero $f$.

E.g. $2+7i$ is a zero of $f$. Then $2-7i$ is also a zero of $f$.

$3 - \frac{1}{2}i$ is a zero of $f$. Then $3 + \frac{1}{2}i$ is also a zero of $f$.