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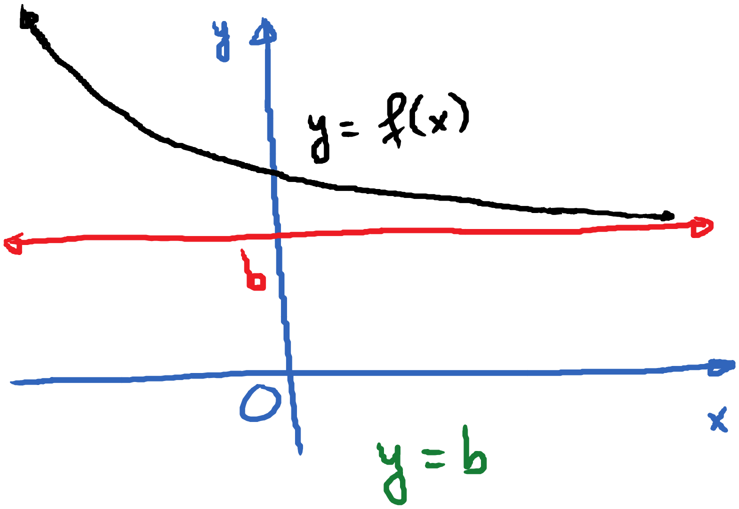
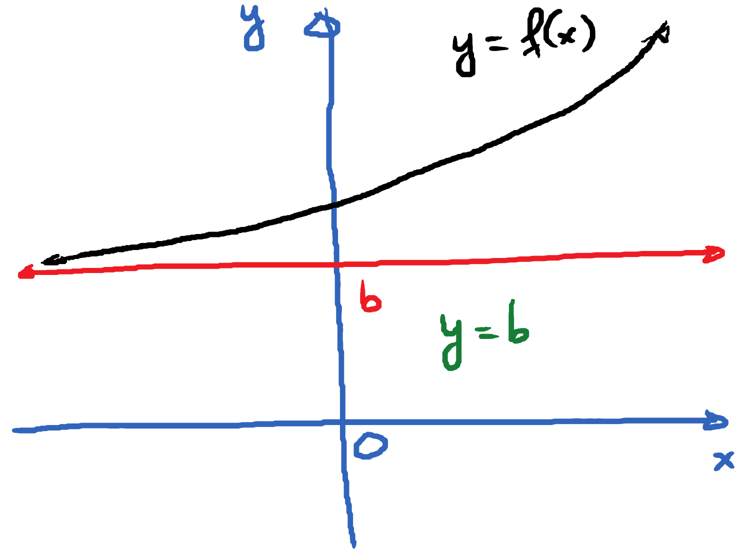
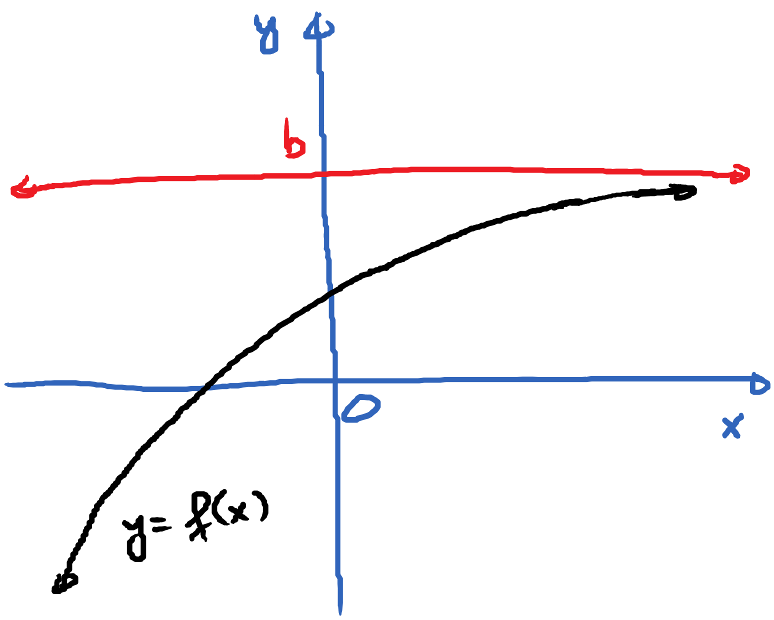
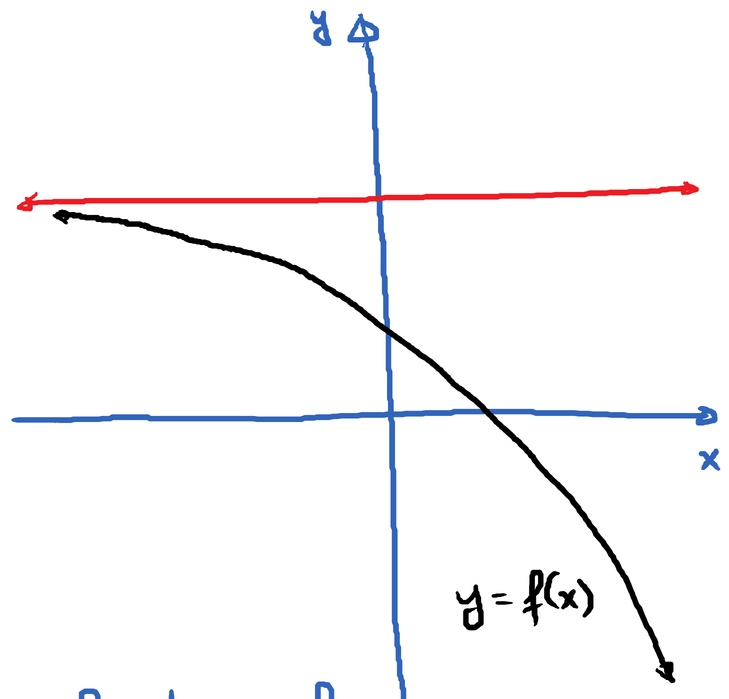
Find horizontal asymptote









We say $y = b$ is a H.A. of the function f if one of the above situations occurs.

Process for finding horizontal asymptote(s) of

$$f(x) = \frac{p(x)}{q(x)}$$

Scenario 1:

$\boxed{\text{Degree top} = \text{Degree bottom}}$

$$\boxed{\text{The H.A. is the line } y = \frac{\text{Leading coeff. of top}}{\text{Leading coeff. of bottom}}}$$

E.g. $f(x) = \dfrac{2x^4 + x^2 + 5}{7x^4 + 3x^2 - 1}$. Find H.A.

Degree top $= 4$; Degree bottom $= 4$

Answer: H.A. $\boxed{y = \frac{2}{7}}$

Scenario 2:

$$\boxed{\text{Degree top} > \text{Degree bottom}}$$

$$\boxed{\text{There is No H.A.}}$$

E.g. $f(x) = \dfrac{3x^5 + 1}{2x^2 + x - 2}$

Degree top $= 5 > 2 =$ Degree bottom

Answer: No H.A.

Scenario 3:

$$\boxed{\text{Degree top} < \text{Degree bottom}}$$

$$\boxed{\text{The H.A. is the line } y = 0}$$

E.g. $f(x) = \dfrac{x^3 - x + 1}{8x^7 + x^4 - 1}$

Degree top $= 3 < 7 =$ Degree bottom

H.A. $\boxed{y = 0}$

E.g. Find the H.A. of the given function:

ⓐ $f(x) = \dfrac{-7x^4 - 10x^2 + 1}{11x^4 + x - 2}$

Ans: $\boxed{y = -\dfrac{7}{11}}$

ⓑ $g(x) = \dfrac{2x + 3}{x^3 - 2x^2 + 4}$

Ans: $\boxed{y = 0}$

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Graph Rational Functions

E.g. Graph: $f(x) = \dfrac{2x+3}{3x^2+7x-6}$.

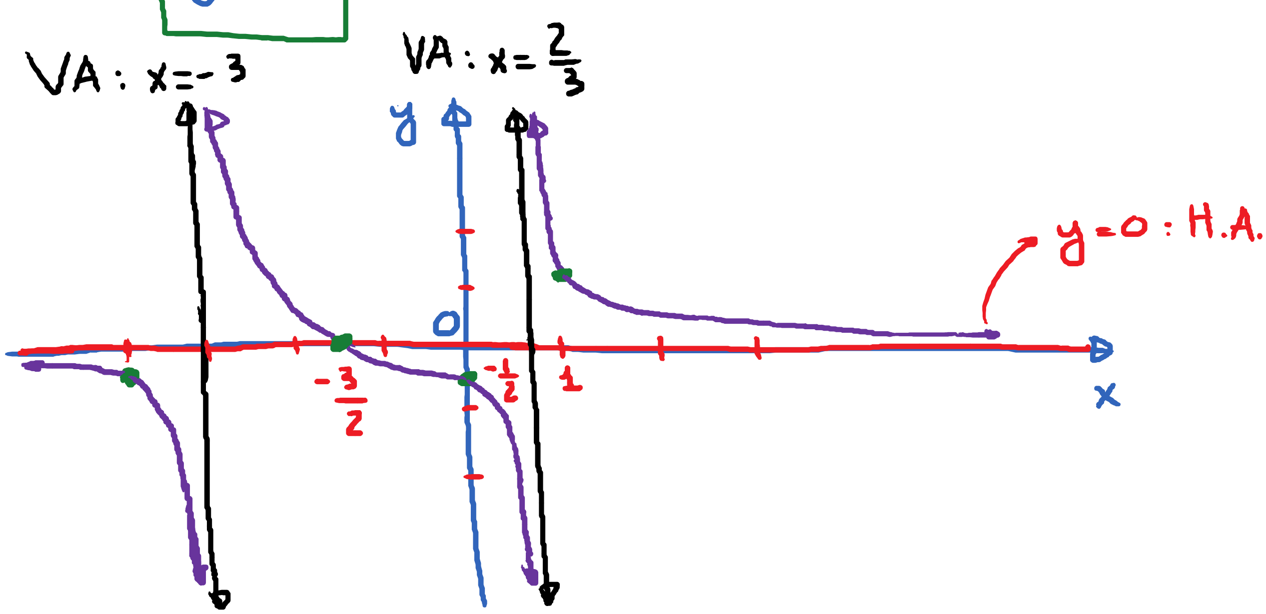
① Find Asymptotes

* V.A.: Factor: $f(x) = \dfrac{2x+3}{(3x-2)(x+3)}$

Solve: $(3x-2)(x+3)=0$

$\boxed{x=\frac{2}{3}}$; $\boxed{x=-3}$ → V.A.

* H.A. $\boxed{y=0}$



(2) Find x-intercept(s) and y-intercept

* x-intercept(s) : Set top = 0

$$2x+3=0 \longleftrightarrow x=-\frac{3}{2}.$$

x-intercept : $\boxed{\left(-\frac{3}{2},0\right)}$

* y-intercept : Set $x=0$ in the formula for $f(x)$

$$f(0)=\frac{2\cdot(0)+3}{3\cdot(0)^2+7\cdot 0-6}=\frac{3}{-6}=-\frac{1}{2}$$

y-intercept : $\boxed{\left(0,-\frac{1}{2}\right)}$

(3) Find additional points (as necessary)

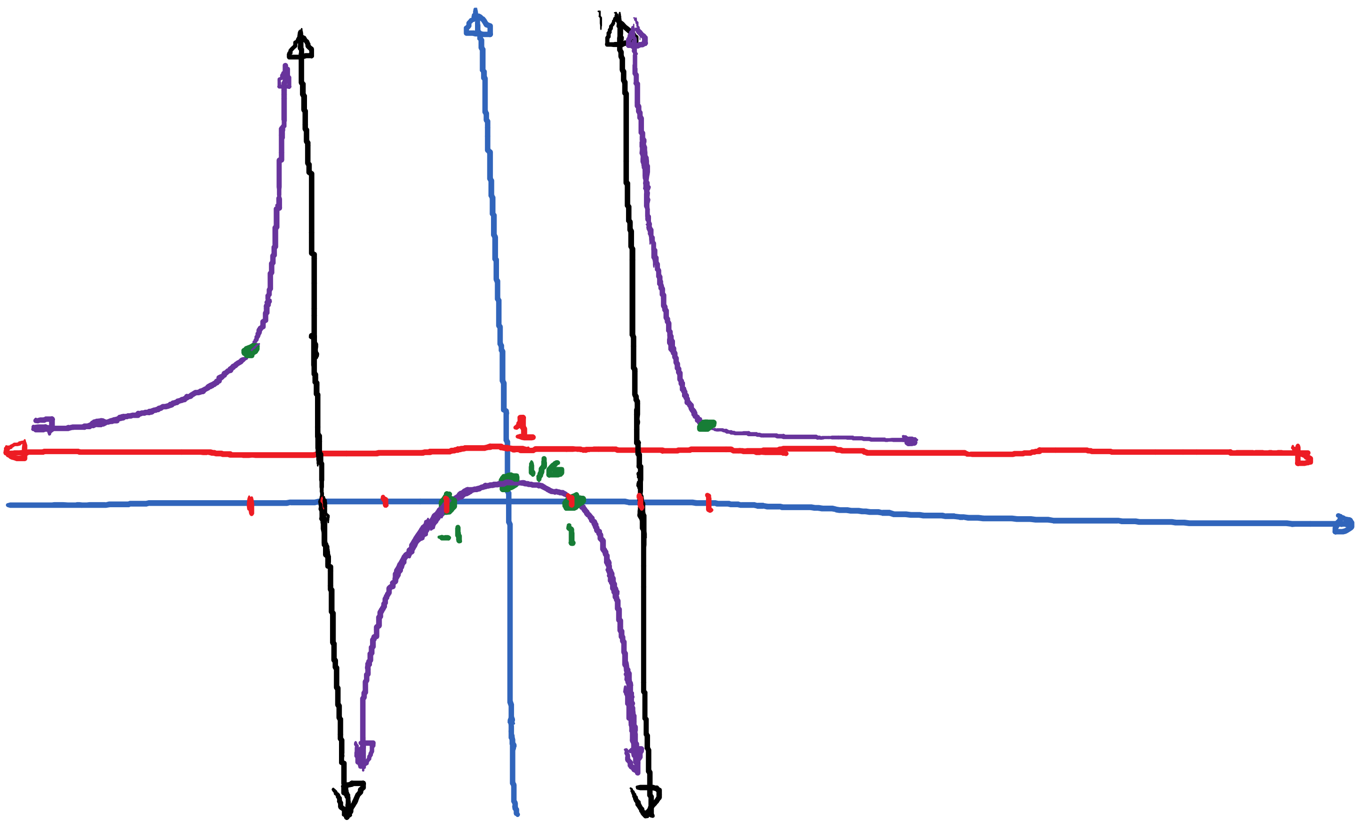
x	$y=\dfrac{2x+3}{3x^2+7x-6}$
1	$\dfrac{2+3}{3+7-6}=\dfrac{5}{4}$
-4	$\dfrac{-8+3}{48-28-6}=\dfrac{-5}{14}$

E.g. Graph: $g(x) = \dfrac{x^2 - 1}{x^2 + x - 6}$

1 Find asymptotes

V.A. $\boxed{x = -3}$; $\boxed{x = 2}$

H.A. $\boxed{y = 1}$



2 x-intercept(s) and y-intercept

x-intercept(s): Set $x^2-1=0$; $x=\pm 1$

$\boxed{(1,0)}$ and $\boxed{(-1,0)}$

y-intercept: $f(0)=\frac{1}{6}$. $\boxed{\left(0,\frac{1}{6}\right)}$

3 Additional points

x	$y=\frac{x^2-1}{x^2+x-6}$
3	$\frac{8}{6}=\frac{4}{3}$
-4	$\frac{15}{6}$