# 8.6. Polynomial Inequalities and Rational Inequalities

Tuesday, November 6, 2018 11:59 AM

Objective: Solve poly. and rational inequalities.

E.g. Solve $\underbrace{x^2 - 4x - 5}_{f(x)} \boxed{> 0}$ → means ⊕

Step 1: Find zeros.

$$x^2 - 4x - 5 = 0$$

$$(x+1)(x-5) = 0 \longrightarrow x = -1;\ x = 5$$

Step 2: Make a sign chart


| Intervals | $(-\infty, -1)$ | $(-1, 5)$ | $(5, \infty)$ |
|---|---|---|---|
| Test value | $f(-2) = 7$ | $f(0) = -5$ | $f(6) = 7$ |
| Sign of $f$ | + | − | + |

Solution set: $\boxed{(-\infty, -1) \cup (5, \infty)}$ or $\boxed{\{x \mid x < -1 \text{ or } x > 5\}}$

E.g. Solve the inequality $x^2+3x-5 \le x+3$.

$$x^2+3x-5-x-3 \le 0$$

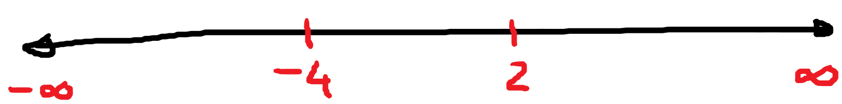
$$\underbrace{x^2+2x-8}_{f(x)} \le 0$$

$\le 0$ → means $-$ or "$=$" to $0$

**Step 1:** Find zeros $(x+4)(x-2)=0$

$x=-4$; $x=2$

**Step 2:** Sign chart




| Intervals | $(-\infty,-4)$ | $(-4,2)$ | $(2,\infty)$ |
|---|---|---|---|
| Test Value | $f(-5)=7$ | $f(0)=-8$ | $f(3)=7$ |
| Sign of f | $+$ | $-$ | $+$ |

Solution set: $\boxed{[-4,2]}$ or $\boxed{\{x \mid -4 \le x \le 2\}}$

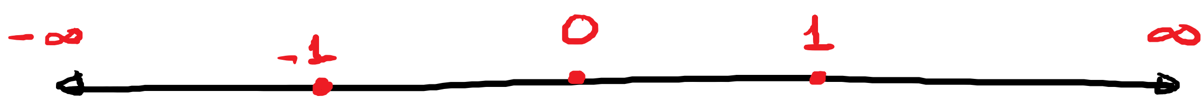
E.g. $\underbrace{x^3 - x}_{f(x)} \geq 0$

**Step 1:** Find zeros of $f$

$$x^3 - x = 0 \longleftrightarrow x(x^2 - 1) = 0$$

$$\longleftrightarrow x(x+1)(x-1) = 0 \longleftrightarrow x = 0;\ x = -1,\ x = 1$$

**Step 2:** Make a sign chart for $f$




| Interval | $(-\infty, -1)$ | $(-1, 0)$ | $(0, 1)$ | $(1, \infty)$ |
|---|---|---|---|---|
| Test Value | $f(-2) = -6$ | $f(-\frac{1}{2}) = \frac{3}{8}$ | $f(\frac{1}{2}) = -\frac{3}{8}$ | $f(2) = 6$ |
| Sign of $f$ | $-$ | $+$ | $-$ | $+$ |

Solution set: $\boxed{[-1, 0] \cup [1, \infty)}$

or $\boxed{\{x \mid -1 \leq x \leq 0 \text{ or } x \geq 1\}}$

# * Rational Inequalities

<u>E.g.</u> Solve the inequality: $\boxed{\underbrace{\frac{3x}{x+6}}_{}} \leq 0$ — $f(x)$

<u>Step 1:</u> Find critical values of $f$ <  Set top $=0$ / Set bottom $=0$

$$\left.\begin{array}{l} \text{Top} = 0 \leftrightarrow 3x = 0 \leftrightarrow x = 0 \\ \text{Bottom} = 0 \leftrightarrow x+6 = 0 \leftrightarrow x = -6 \end{array}\right\} \text{critical values of } f$$

<u>Step 2:</u> Make a sign chart for $f$


| Interval | $(-\infty,-6)$ | $(-6,0)$ | $(0,\infty)$ |
|---|---|---|---|
| Test Value | $f(-7)=21$ | $f(-1)=\frac{-3}{5}$ | $f(1)=\frac{3}{7}$ |
| Sign of $f$ | $+$ | $-$ | $+$ |

Solution: $\boxed{(-6,0]}$ or $\boxed{\{x \mid -6 < x \leq 0\}}$

E.g. Solve $\dfrac{x+1}{2x-4} \le 1$

$$\frac{x+1}{2x-4} - \frac{1\cdot(2x-4)}{1\cdot(2x-4)} \le 0$$

$$\frac{x+1}{2x-4} - \frac{2x-4}{2x-4} \le 0$$

$$\frac{x+1-(2x-4)}{2x-4} \le 0$$

$$\frac{x+1-2x+4}{2x-4} \le 0$$

$$\underbrace{\boxed{\frac{-x+5}{2x-4}}}_{f(x)} \le 0$$

Critical Values: $x=5$ ; $x=2$

| Interval | $(-\infty, 2)$ | $(2, 5)$ | $(5, \infty)$ |
|---|---|---|---|
| Test Value | $f(0) = -\frac{5}{4}$ | $f(3) = 1$ | $f(6) = -\frac{1}{8}$ |
| Sign | $-$ | $+$ | $-$ |

Solution set: $\boxed{(-\infty, 2) \cup [5, \infty)}$

or $\boxed{\{x \mid x < 2 \text{ or } x \geq 5\}}$