E.g. $h(x) = \dfrac{2}{(x+3)^3} + 5$

Find f and g such that $f \circ g = h$.

* $f(x) = \dfrac{2}{x^3} + 5$; $g(x) = x+3$.

* $f(x) = \dfrac{2}{(x+1)^3} + 5$; $g(x) = x+2$

* $f(x) = 2x+5$, $g(x) = \dfrac{1}{(x+3)^3}$

* Domain of a composite function.

E.g. $f(x) = \dfrac{4}{x+2}$; $g(x) = \dfrac{1}{x}$.

We saw that $\boxed{(f \circ g)(x)} = f(g(x)) = \dfrac{4x}{1+2x}$.

Q: Find the domain of $f \circ g$?

Tuesday, November 13, 2018 11:05 AM

Step 1: Find the domain of the inside function.

(Find the domain of g in this example)

$g(x) = \frac{1}{x}$. Domain? $D_g = \{x \mid x \neq 0\}$

Step 2: Find the domain of the formula for $f(g(x))$

$f(g(x)) = \frac{4x}{1+2x}$. Domain?

$1 + 2x = 0 \longleftrightarrow x = -\frac{1}{2}$. Domain $= \{x \mid x \neq -\frac{1}{2}\}$

Step 3: Domain of the composite function $f \circ g$

$D_{f \circ g}$ = Intersection of Step 2 and Step 3

= all real numbers except for 0 and $-\frac{1}{2}$.

$= \boxed{\{x \mid x \neq 0 \text{ and } x \neq -\frac{1}{2}\}}$

or $= \boxed{(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)}$

Strategy to find the domain of the composite function $f \circ g$.

Step 1: Find domain of the inside; i.e., find D_g

Step 2: Find domain of the formula for $f(g(x))$.

Step 3: Answer = Intersection of Step 1 and Step 2.

Ex. Given $f(x) = \dfrac{2}{x+3}$; $g(x) = \dfrac{1}{x-1}$.

Find the domain of $f \circ g$.

Step 1: $D_g = \{x \mid x \neq 1\}$

Step 2: $f(g(x)) = f\left(\dfrac{1}{x-1}\right) = \dfrac{2}{\dfrac{1}{x-1} + \dfrac{3\cdot(x-1)}{1\cdot(x-1)}}$

$$= \frac{2}{\dfrac{1}{x-1} + \dfrac{3(x-1)}{x-1}} = \frac{2}{\dfrac{1+3x-3}{x-1}}$$

$$= \frac{2}{\dfrac{3x-2}{x-1}} = \frac{2}{1} \cdot \frac{x-1}{3x-2} = \boxed{\frac{2(x-1)}{3x-2}}$$

$$3x - 2 = 0 \longleftrightarrow x = \frac{2}{3}$$

Domain of $f(g(x))$: $\left\{ x \mid x \neq \frac{2}{3} \right\}$

Step 3: $D_{f \circ g} = \boxed{\left\{ x \mid x \neq 1 \text{ and } x \neq \frac{2}{3} \right\}}$

$$= \boxed{\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 1\right) \cup (1, \infty)}$$