# 9.1 The composition of Functions

Thursday, November 8, 2018 11:27 AM

**Objectives:** (1) Find the composition of functions

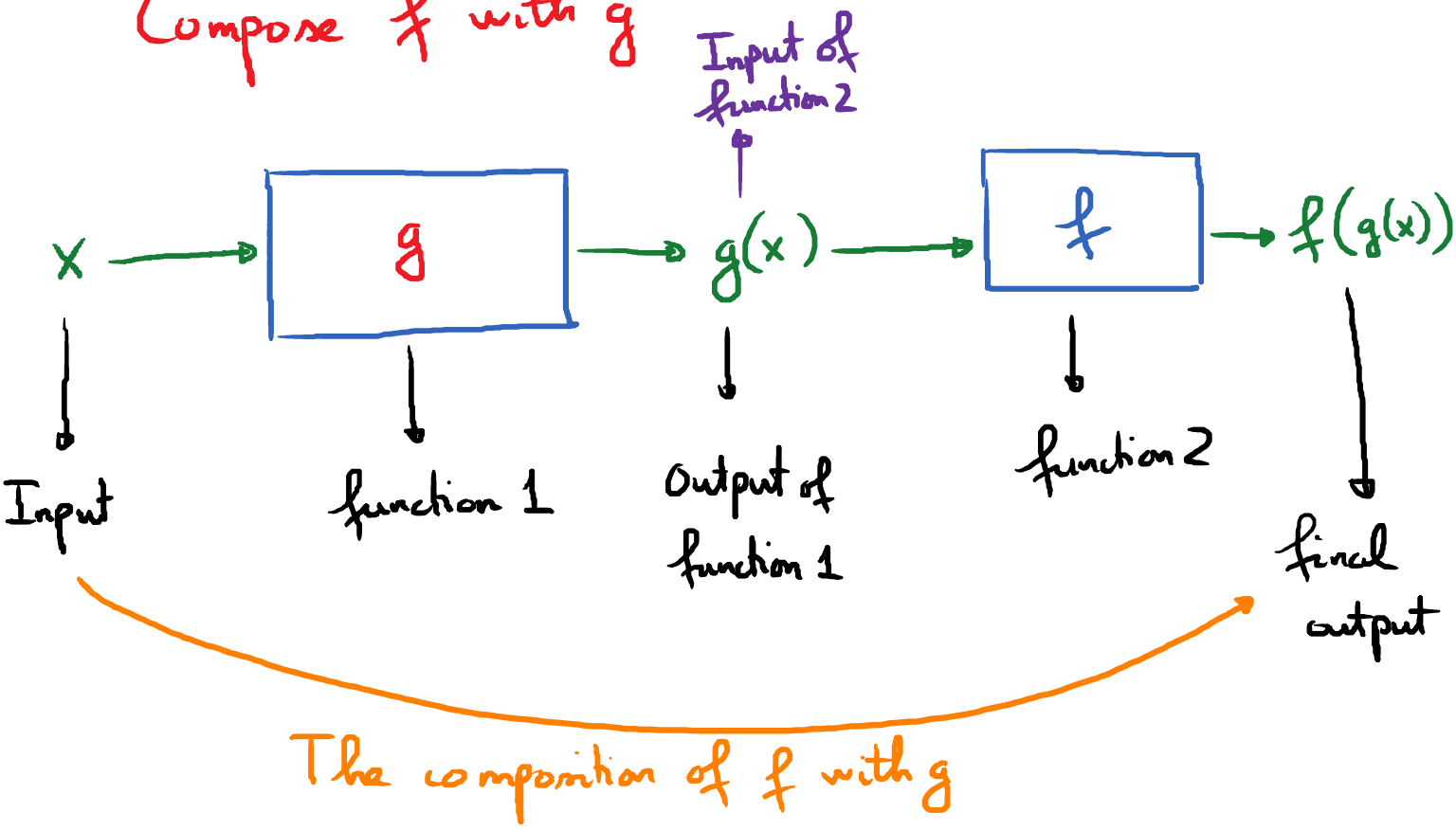
(2) Find the domain of compositions of functions

(3) Decompose Functions.

## (1) Composing Functions

E.g. $f(x) = 2x - 5$ ; $g(x) = x^2 - 3x + 8$

Compose $f$ with $g$



$$f(g(x)) = f(x^2 - 3x + 8)$$

$$= 2(x^2 - 3x + 8) - 5$$

$$= 2(x^2 - 3x + 8) - 5$$

$$= 2x^2 - 6x + 16 - 5$$

$$f(g(x)) = 2x^2 - 6x + 11$$

read as f of g of x

* Find $g(f(x))$

$$g(f(x)) = g(2x - 5)$$

$$= (2x - 5)^2 - 3(2x - 5) + 8$$

$$= (2x - 5)(2x - 5) - 6x + 15 + 8$$

$$= 4x^2 - 10x - 10x + 25 - 6x + 23$$

$$= 4x^2 - 26x + 48$$

# Composition of Functions $f \circ g$

The composite function $f \circ g$ is defined to be:

$$f \circ g(x) = f(g(x))$$

$x$ is in $D_g$

$g(x)$ is in $D_f$

Similarly,

$$g \circ f(x) = g(f(x))$$

$x$ is in $D_f$

$f(x)$ is in $D_g$

Ex (a) Given $f(x) = \frac{4}{x+2}$ ; $g(x) = \frac{1}{x}$

Find and simplify $f \circ g$ and $g \circ f$

(b) Given $f(x) = \sqrt{x} + 1$ ; $g(x) = x - 3$.

Find and simplify $f \circ g$, $g \circ f$; $g \circ g$; $f \circ f$.

$f \circ g$: $\sqrt{x-3} + 1$

$g \circ f$: $\sqrt{x} - 2$

$g \circ g$: $x - 6$

$f \circ f$: $\sqrt{\sqrt{x} + 1} + 1$

Sol

(a) $f \circ g(x) = f(g(x))$

$$= \frac{4}{\frac{1}{x} + \frac{2 \cdot x}{1 \cdot x}} = \frac{4}{\frac{1}{x} + \frac{2x}{x}}$$

$$= \frac{4}{\frac{1+2x}{x}} = \frac{4}{1} \cdot \frac{x}{1+2x}$$

$$\boxed{f \circ g(x) = \frac{4x}{1+2x}}$$

(b) $(g \circ f)(x) = g(f(x))$

$$= \frac{1}{\frac{4}{x+2}} = \frac{1}{1} \cdot \frac{x+2}{4} = \boxed{\frac{x+2}{4}}$$

# (3) Decomposing functions

**E.g.** Given $h(x) = (2x-3)^5$

**Q:** Find $f$ and $g$ such that $f \circ g$ ?

$$f(x) = x^5 \; ; \; g(x) = 2x-3$$

**Check:** $f \circ g(x) = f(g(x))$

$$= f(2x-3)$$

$$= (2x-3)^5$$

Another answer: $f(x) = (x+8)^5$ ; $g(x) = 2x-11$.

**Check:** $f \circ g(x) = f(g(x)) = f(2x-11)$

$$= (2x-11+8)^5 = (2x-3)^5 .$$