Step 4: Replace "y" by "$f^{-1}(x)$"

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$$

→ this is the formula for the inverse function of f.

E.x. Apply this process to find the inverse function of

$$f(x) = \frac{5}{x} - 6.$$

Step 1: $y = \frac{5}{x} - 6$ (Replace $f(x)$ by y)

Step 2: Get x by itself:

$$y + 6 = \frac{5}{x}$$

→ Multiply both sides by x

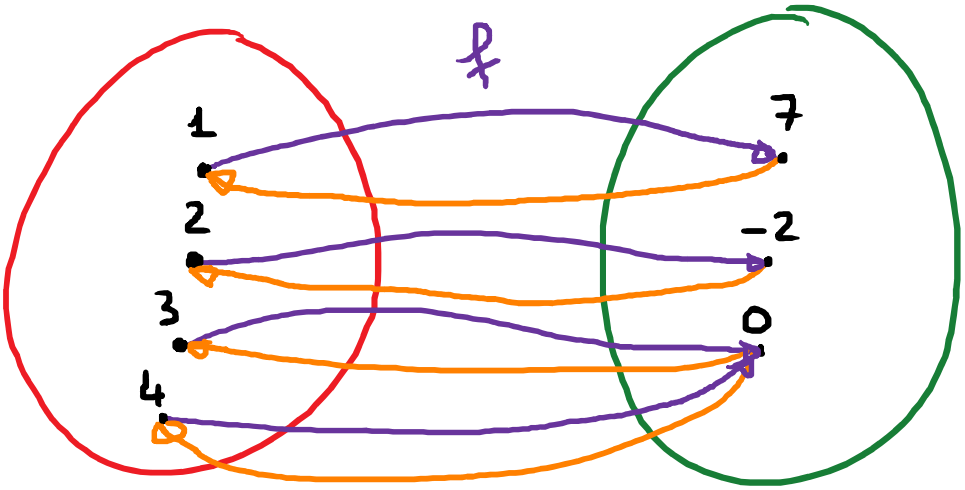
$$x \cdot (y+6) = 5$$

$$x = \frac{5}{y+6}$$

Step 3: $y = \dfrac{5}{x+6}$ (Switch "x" and "y")

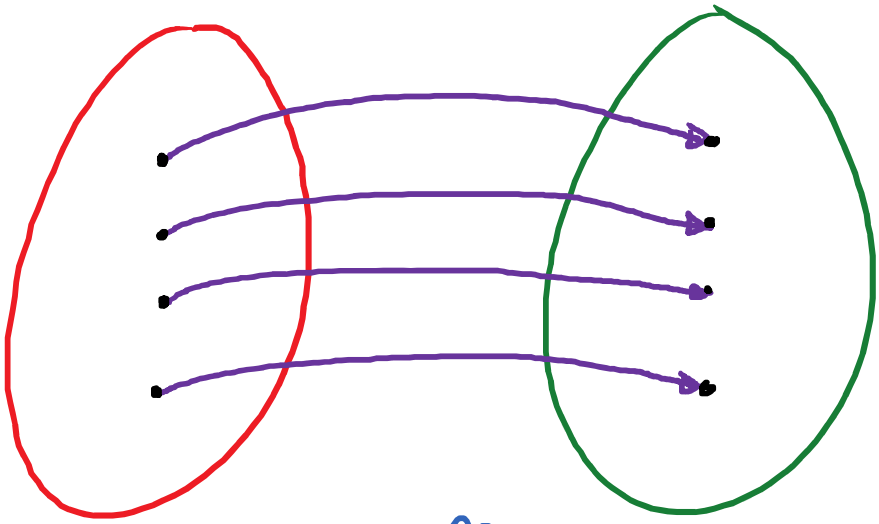
Step 4: $\boxed{f^{-1}(x) = \dfrac{5}{x+6}}$ (Replace "y" by "$f^{-1}(x)$")

(3) One-to-one functions and horizontal line test



f^{-1} → NOT a function

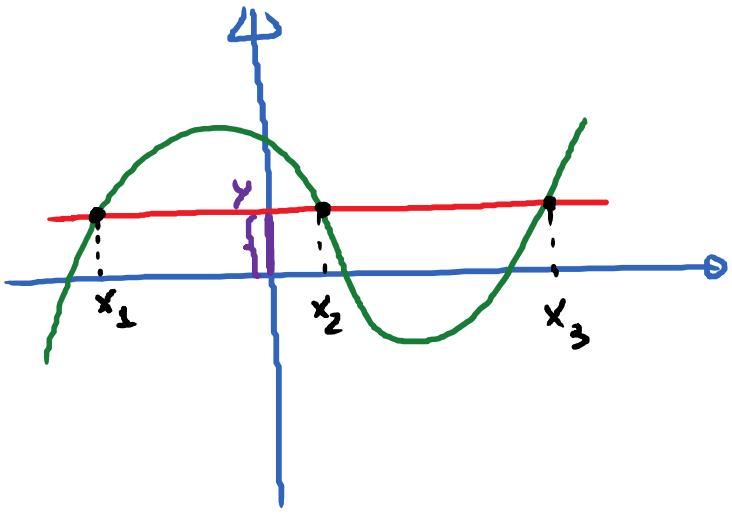
For a function to have an inverse function it must look like this



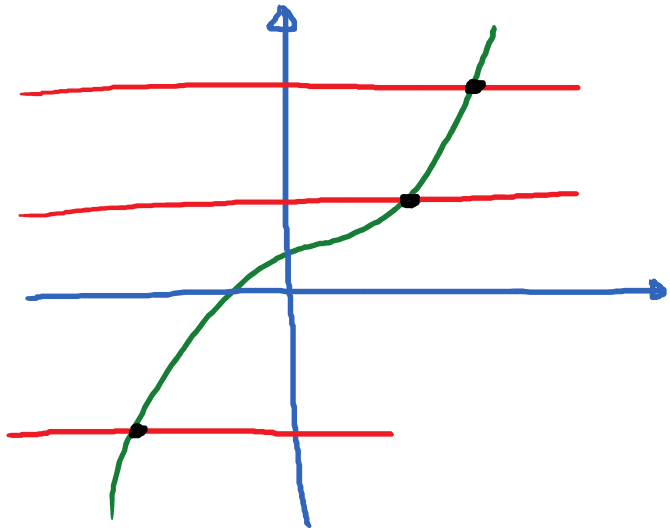
This is called a one-to-one function

A function is one-to-one if every output corresponds to exactly one input.

→ Horizontal line Test for one-to-one function.



Not one-to-one



one-to-one.

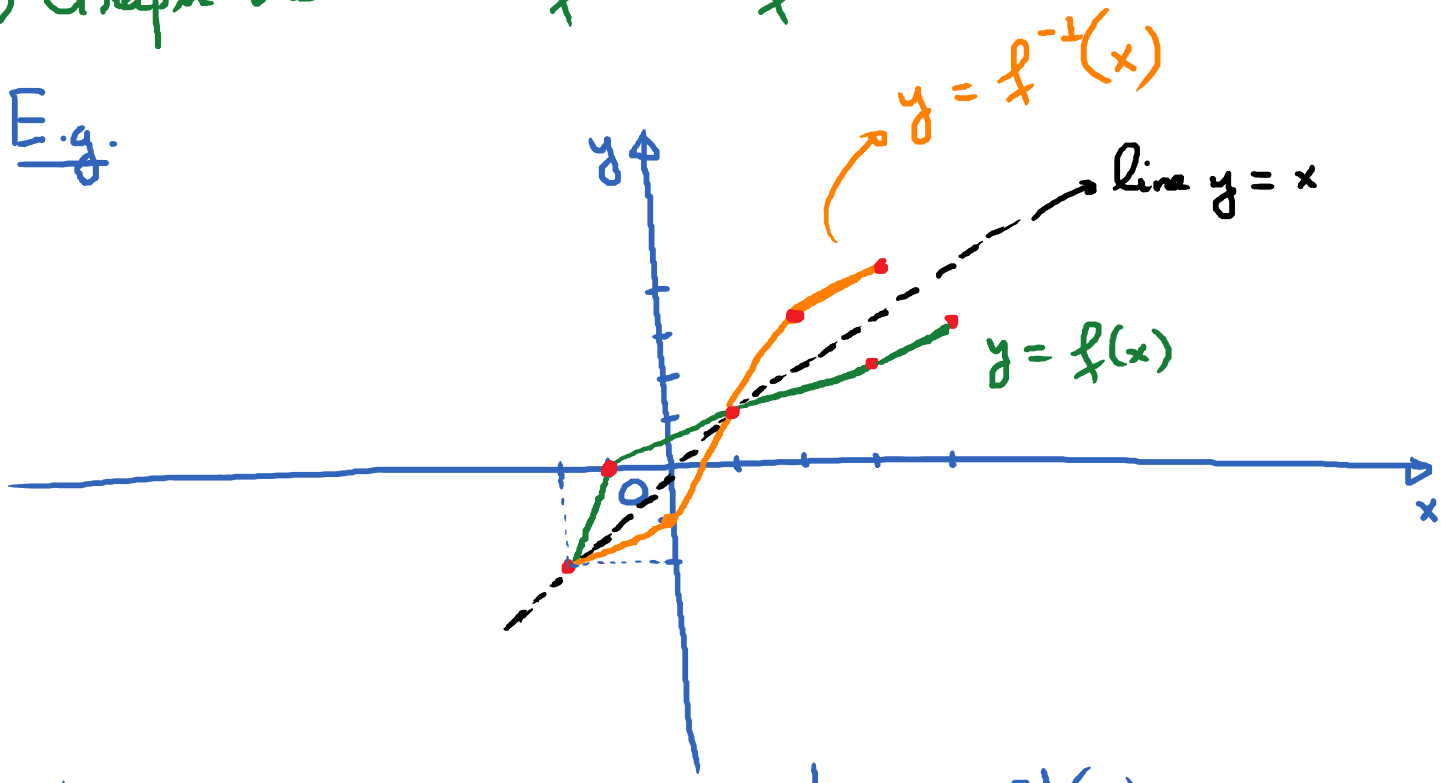
Horizontal Line Test:

If there is a horizontal line which intersects a graph at more than one points, then the graph is NOT one-to-one.

If every horizontal line intersects a graph exactly once, the the graph is one-to-one.

(4) Graph the inverse function of a one-to-one function.

E.g.



x	$y = f(x)$
-2	-2
-1	0
1	1
3	2
4	3

x	$y = f^{-1}(x)$	
-2	-2	(-2, -2)
0	-1	(0, -1)
1	1	(1, 1)
2	3	(2, 3)
3	4	(3, 4)

Observation: Graph of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric with respect to the line $y = x$