9.2. Inverse Functions

Tuesday, November 13, 2018 11:25 AM

Objectives: (1) Define inverse relations and inverse functions and verify that 2 functions are inverses of one another

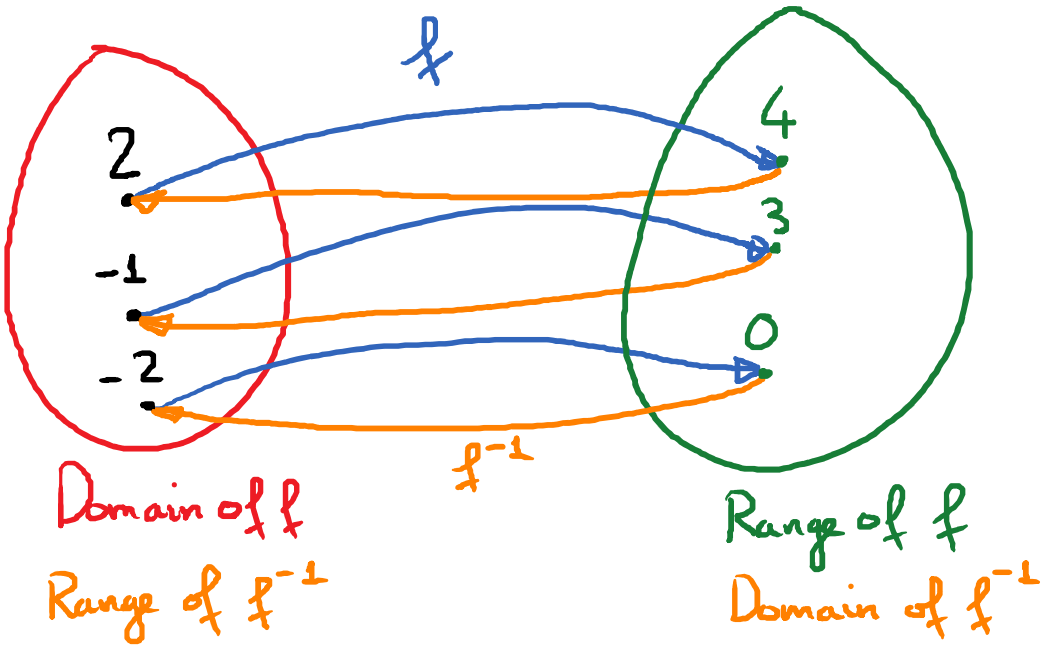
(2) Find the inverse of a given function.

(3) One-to-one functions and horizontal line test.

(4) Graphs of a function and its inverse.

Inverse Relation

Given a relation $f = \{(2,4); (-1,3); (-2,0)\}$

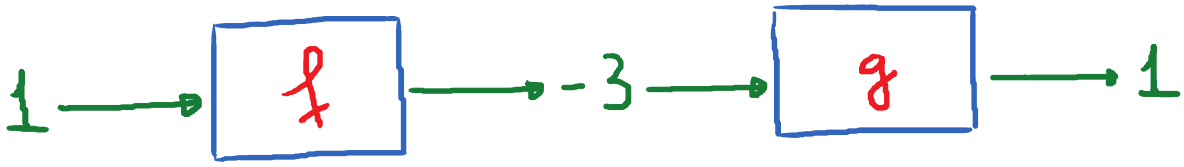


The inverse relation of f; denoted by f^{-1}, is the relation:

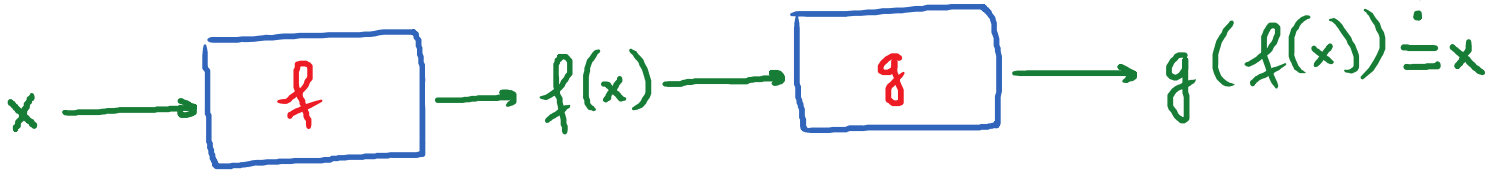
$$f^{-1} = \{(4,2);(3,-1);(0,-2)\}$$

Inverse Functions:

E.g. $f(x) = 4x - 7$; $g(x) = \frac{x+7}{4}$.



Is it true that:



We say that functions f and g are inverses of each other if:

(1) $(f \circ g)(x) = f(g(x)) = x$ for x in D_g

(2) $(g \circ f)(x) = g(f(x)) = x$ for x in D_f.

E.g. Demonstrate that $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$ are inverses of each other.

$$f(g(x)) = f\left(\frac{x+7}{4}\right) = \frac{4}{1}\cdot\left(\frac{x+7}{4}\right) - 7$$

$$= x + 7 - 7 = x$$

$$g(f(x)) = g(4x-7) = \frac{(4x-7)+7}{4} = \frac{4x}{4} = x$$

Conclusion: f and g are inverses of one another.

Note: We usually denote the function g by f^{-1} (read as f inverse)

* f^{-1} means the inverse function of f. It does **not** mean $\frac{1}{f}$

Ex. Determine whether $f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$ are inverses of one another.

Sol: $f(g(x)) = f\left(\frac{3}{x}+4\right) = \frac{3}{\left(\frac{3}{x}+4\right)-4}$

$= \frac{3}{\frac{3}{x}} = \frac{3}{1} \cdot \frac{x}{3} = \frac{3x}{3} = x$

$g(f(x)) = g\left(\frac{3}{x-4}\right) = \frac{3}{\frac{3}{x-4}} + 4$

$= \frac{3}{1} \cdot \frac{x-4}{3} + 4$

$= x - 4 + 4 = x$

Yes, they are inverses.

(2) Find the inverse function of a given function

E.g. Find the inverse function of $f(x) = 4x^3 - 1$.

Step 1: Replace "$f(x)$" by "y" in the equation for $f(x)$

$$y = 4x^3 - 1$$

Step 2: Solve for x in terms of y; i.e., get x by itself from the equation in Step 1.

$$\longrightarrow \quad y + 1 = 4x^3$$

$$\longrightarrow \quad \frac{y+1}{4} = x^3$$

$$\longrightarrow \quad \sqrt[3]{\frac{y+1}{4}} = x$$

So, $\boxed{x = \sqrt[3]{\frac{y+1}{4}}}$

Step 3: Interchange the letter "x" and "y" in the equation obtained in Step 2.

$$y = \sqrt[3]{\frac{x+1}{4}}$$