(4) Logarithmic function.

We define the logarithmic function

$y = \log_a x$ as $\log_a x$ gives us the exponent y such that $a^y = x$.

$\log_a x$ read as "log base a of x"

E.g. * Base $a = 2$. $f(x) = \log_2 x$.

$f(8) = \log_2(8) = 3$; $f(4) = \log_2(4) = 2$

$f(1) = \log_2(1) = 0$; $f(16) = \log_2(16) = 4$

* Base $a = 3$. $f(x) = \log_3 x$

$f(9) = \log_3(9) = 2$; $f(1) = \log_3(1) = 0$

$f(3) = \log_3(3) = 1$.

* Base 10 $f(x) = \log_{10} x$

$f(100) = \log_{10}(100) = 2$

$f(1000) = \log_{10}(1000) = 3$

$f(10000) = \log_{10}(10000) = 4$

$f(1,000,000) = \log_{10}(1000000) = 6$

Note:

$\log_a 1 = 0$ for any base a

$\log_a a = 1$ for any base a

Note: $y = \log_a x$; x must be positive for the function to be defined.

Note: $y = \log_a x$ equivalent to $a^y = x$

$y = \log_a x$: logarithmic form

$a^y = x$: exponential form

E.g. (1) Convert to logarithmic form:

(a) $16 = 2^x$ → $\log_2 16 = x$

(b) $10^{-3} = 0.001$ → $\log_{10}(0.001) = -3$

(c) $e^t = 70$ → $\log_e(70) = t$

(2) Convert to exponential form.

(a) $\log_2 32 = 5$ → $2^5 = 32$

(b) $\log_a Q = 8$ → $a^8 = Q$

(c) $x = \log_t M$ → $t^x = M$

Special Notations:

Common Logarithm: $\log_{10} x = \log x$ (read as log of x and it means base 10)

Natural Logarithm: $\log_e x = \ln x$ (read as ln of x and it means base e.)

E.g. $\log(100) = 2$; $\log(1) = 0$

$\ln(1) = 0$; $\ln(e) = 1$

Change of base formula:

Calculate $\log_2 3$

Change to base e: $\frac{\ln(3)}{\ln(2)}$

Change to base 10: $\frac{\log(3)}{\log(2)}$

≈ 1.585

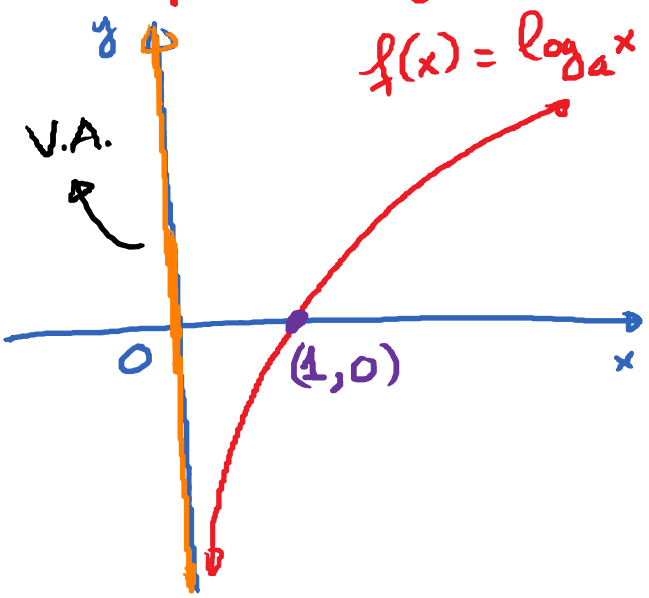
In general,

$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

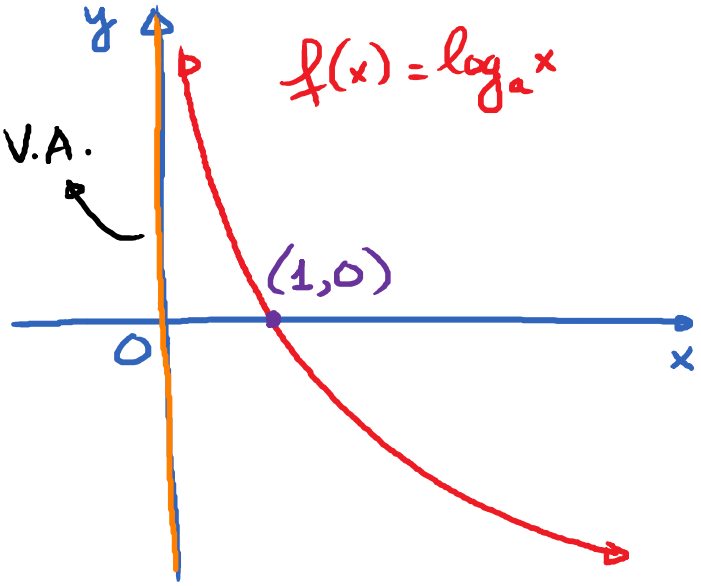
→ change of base

For e.g. $\log_2(3) = \dfrac{\log_7(3)}{\log_7(2)}$

Graphs of Log functions:



$a > 1$



$0 < a < 1$