9.3 and 9.4. Exponential and Logarithmic Functions

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Objectives: (1) Exponential and Logarithmic functions and their graphs.

(2) Base e. $e \approx 2.71828\ldots$

(3) Applications.

(1) Exponential Functions

read as a raised to the x

The function $f(x) = a^x$ where a is a constant, (a is the base)

$a > 0$ and $a \neq 1$ is called the exponential function with base a

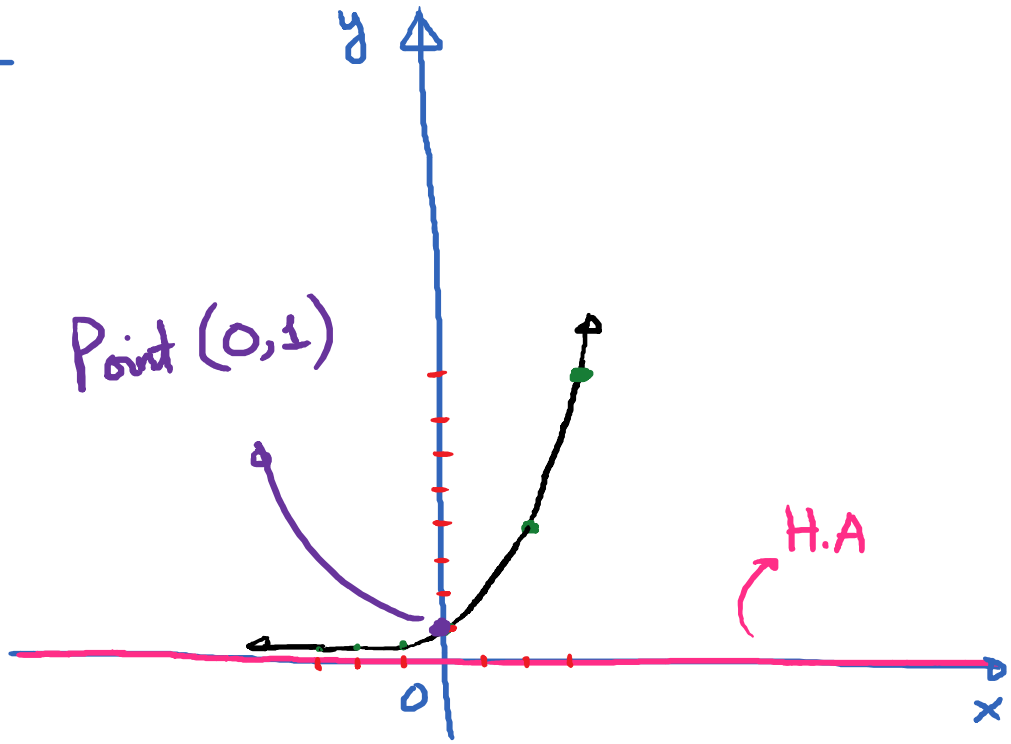
E.g. $f(x) = 2^x$, $g(x) = \left(\frac{1}{2}\right)^x$

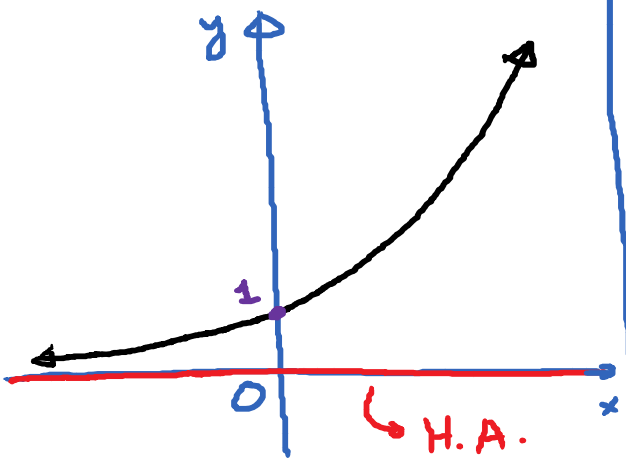
$f(x) = 3^x$, $g(x) = \left(\frac{2}{5}\right)^x$

are exponential functions.

E.g. Graph of $f(x) = 2^x$ (Reminder $2^{-n} = \frac{1}{2^n}$)

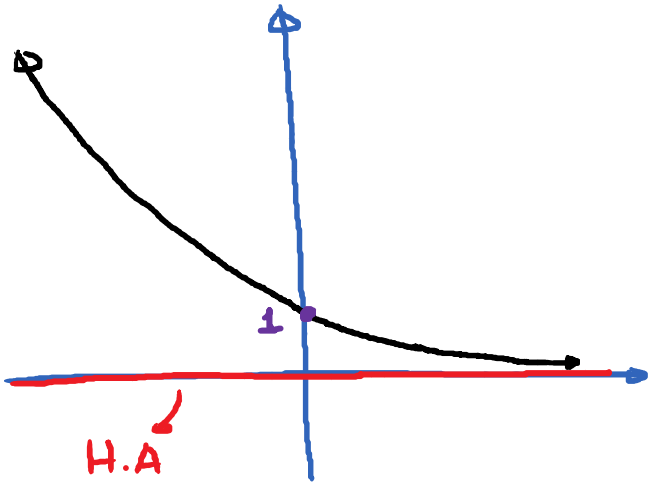
x	$f(x) = 2^x$
0	1
1	2
2	4
3	8
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$
-3	$\frac{1}{8}$





$f(x) = a^x$; $a > 1$

Exponential Growth



$f(x) = a^x$; $0 < a < 1$

Exponential Decay

E.g. Parent function $f(x) = 2^x$.

(1) How do we obtain $y = \underbrace{2^{x-2}}_{f(x-2)}$: Shift to the right 2 units

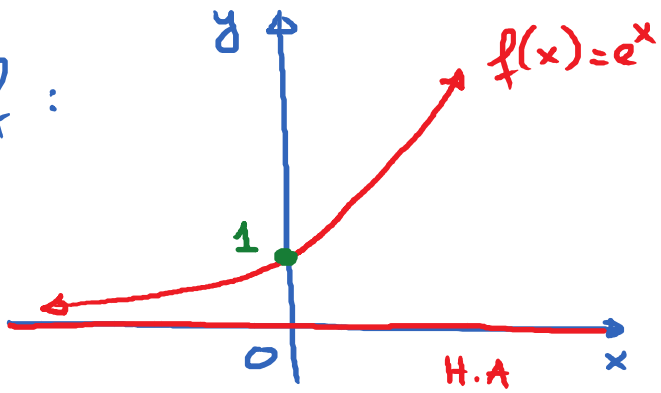
(2) How do we obtain $y = \underbrace{2^x + 4}_{f(x)+4}$: Shift up 4 units

(3) How do we obtain $y = \underbrace{7 - 2^x}_{-f(x)+7}$: Flip across x-axis, up 7.

(2) Exponential Function of base e.

$e \approx 2.71828\cdots$

$f(x) = e^x$. Graph of f :



Thursday, November 15, 2018 11:36 AM

(3) Some applications of Exponential Functions

(1) Compound Interest

$$A = P\left(1 + \frac{R}{n}\right)^{nt}$$

A: Final amount in account after t years

P: Principal

t: # of years

n: # of compounding periods a year

R: yearly interest rate

(2) Continuous compound interest

$$A = P \cdot e^{Rt}$$

A: Final amount after t years

P: Principal

t: # of years

Here interest is calculated continuously.

R: yearly interest rate.

E.g. Suppose that $10000 is invested in an account that pays 6.5% interest. Find the final amount after 10 years if

(a) It is compounded semiannually

(b) It is compounded quarterly

(c) It is compounded continuously.

(a) $A = \$10000\left(1 + \frac{0.065}{2}\right)^{2 \cdot 10} \approx \$18958.38$

(b) $A = \$10000\left(1 + \frac{0.065}{4}\right)^{4 \cdot 10} \approx \$19055.59$

(c) $A = \$10000 \cdot e^{(0.065)(10)} \approx \$19155.41$