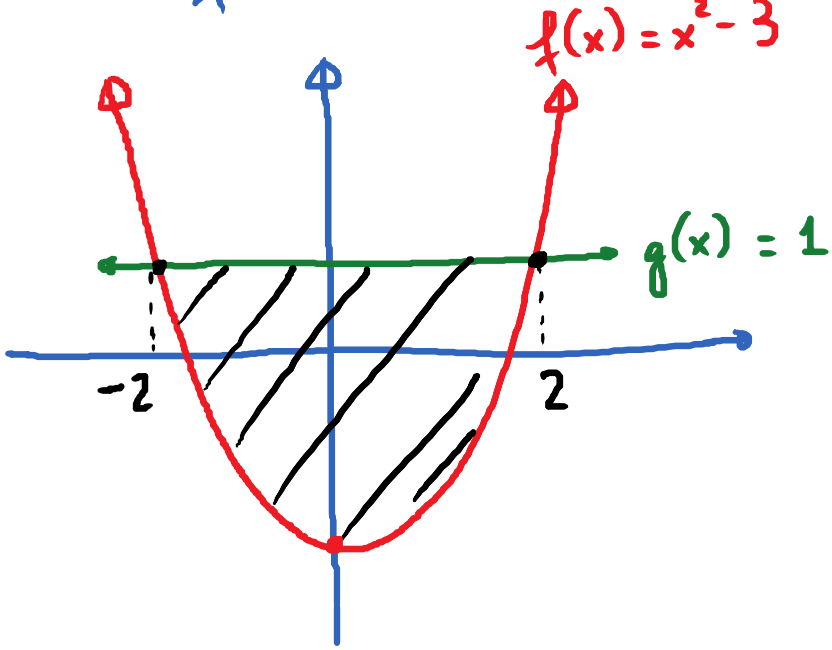
E.g. Find the area of the region bounded by the 2 curves $f(x) = x^2 - 3$ and $g(x) = 1$.



Step 1: x-coordinates of points of intersection:

Set $f(x) = g(x)$

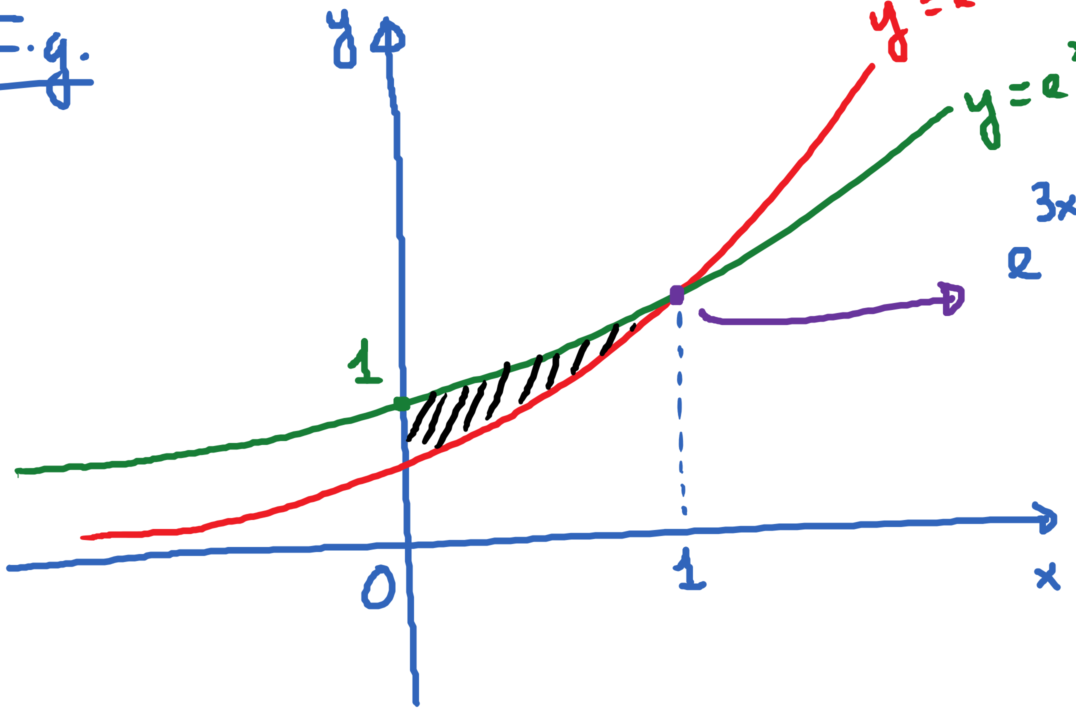
$$x^2 - 3 = 1 \longrightarrow x^2 = 4 \rightarrow x = \pm 2$$

Step 2: Area $= \displaystyle\int_{-2}^{2} \left(1 - (x^2 - 3)\right) dx$

$$= \int_{-2}^{2} (4 - x^2)\, dx = \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{2}$$

$$= \left(4(2) - \frac{8}{3}\right) - \left(4(-2) + \frac{8}{3}\right) = \boxed{\frac{32}{3}}$$

E.g.



$$e^{3x-2} = e^x \to \boxed{x = 1}$$

$$\frac{2}{3}e + \frac{1}{3}e^{-2} - 1$$

Find the area of the shaded region bounded by $y = e^x$ and $y = e^{3x-2}$.

$$\text{Area} = \int_0^1 \left(e^x - e^{3x-2}\right) dx$$

Let $u = 3x - 2$

$du = 3\,dx$

$dx = \frac{du}{3}$

$$= \int_0^1 e^x\,dx - \int_0^1 e^{3x-2}\,dx$$

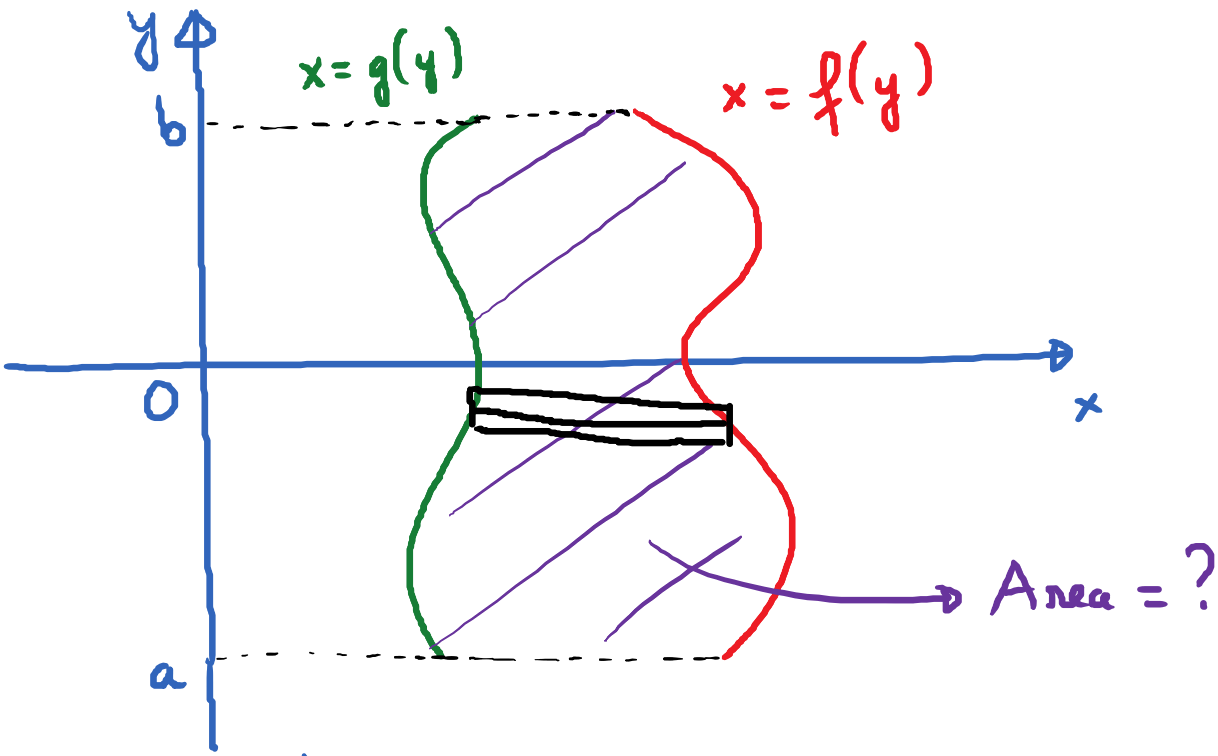
$$= e^x \Big|_0^1 - \frac{1}{3}\int_{-2}^{1} e^u\,du$$

$$= e - 1 - \frac{1}{3} e^u \Big|_{-2}^{1}$$

$$= e - 1 - \frac{1}{3}\left(e - e^{-2}\right)$$

$$= e - 1 - \frac{1}{3}e + \frac{1}{3}e^{-2} = \boxed{\frac{2}{3}e - 1 + \frac{1}{3}e^{-2}}$$

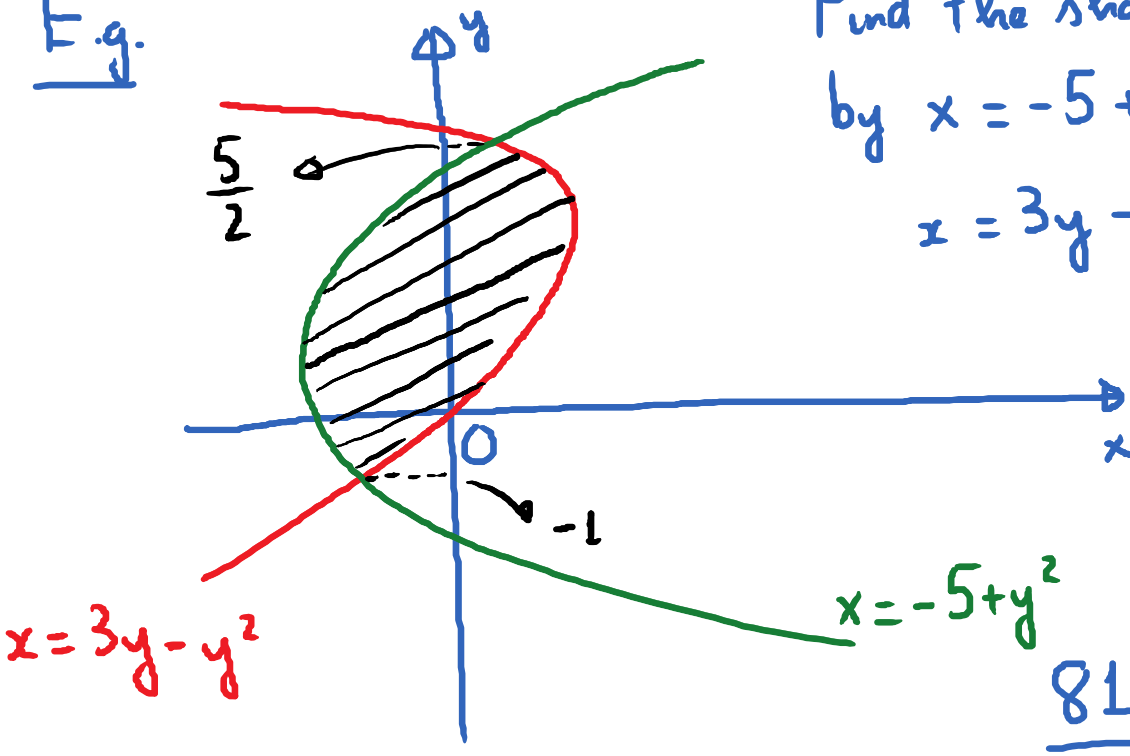
Integration with respect to y.



$$\int_a^b \Big[\underbrace{f(y)}_{\text{rightmost}} - \underbrace{g(y)}_{\text{leftmost}} \Big] \, dy$$

E.g.

Find the shaded area bounded by $x = -5 + y^2$ and $x = 3y - y^2$



$\frac{125}{8}$

$\frac{81}{8}$; 15.625

Step 1: Find pts of intersection:

$$3y - y^2 = -5 + y^2$$

$$2y^2 - 3y - 5 = 0$$

$$(2y - 5)(y + 1) = 0$$

$$y = \frac{5}{2} \; ; \; y = -1$$

$$\int_{-1}^{\frac{5}{2}} \left[\left(3y - y^2\right) - \left(-5 + y^2\right) \right] dy$$

$$= \int_{-1}^{\frac{5}{2}} \left(-2y^2 + 3y + 5\right) dy$$

$$= \left(-2\frac{y^3}{3} + 3\frac{y^2}{2} + 5y\right) \Bigg|_{-1}^{\frac{5}{2}}$$

$$= \boxed{\frac{343}{24}}$$