2.1. Areas

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Recall:

Basic Antiderivatives

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \; ; \; n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \; ; \; \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \; ; \; \int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int e^x\,dx = e^x + C \; ; \; \int \frac{dx}{1+x^2} = \arctan(x) + C$$

u - substitution

$$\int \tan(x)\,dx = -\int \frac{-\sin(x)}{\cos(x)}\,dx = -\int \frac{du}{u}$$

$\cos(x) \to u$, $-\sin(x)\,dx \to du$

Let $u = \cos(x)$; $du = -\sin(x)\,dx$

$$-\ln|u| + C = -\ln|\cos(x)| + C$$

$$= \boxed{\ln|\sec(x)| + C}$$

E.g. $\int e^x \cdot \cos(e^x)\,dx$

$e^x \to u$, $e^x\,dx \to du$

Let $u = e^x$. Then $du = e^x\,dx$

$$\int \cos(u)\,du = \sin(u) + C$$

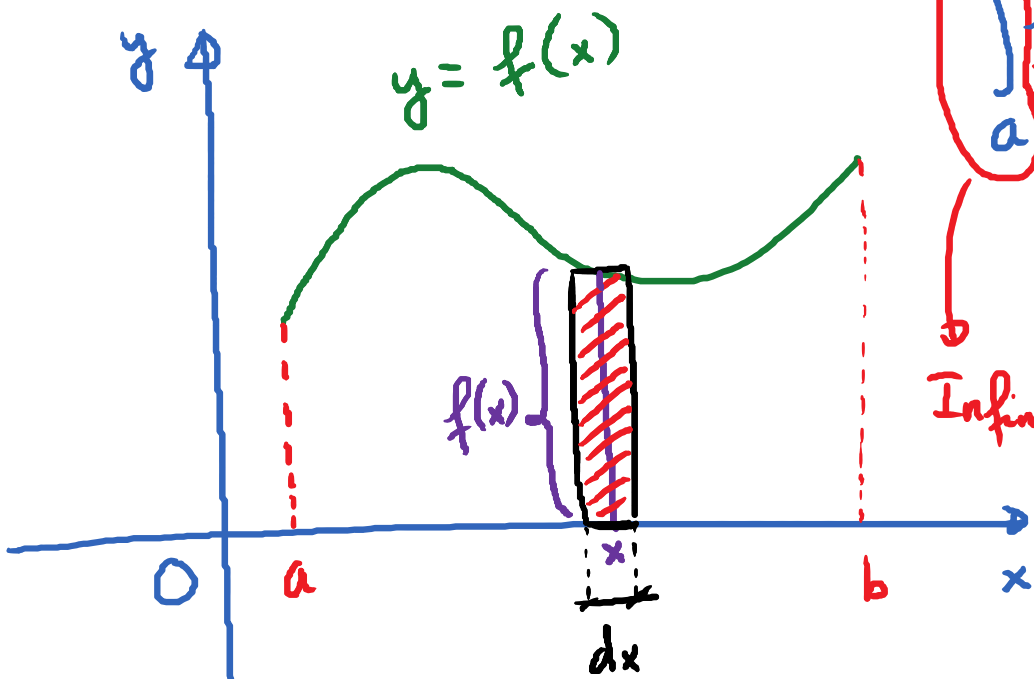
$$= \boxed{\sin(e^x) + C}$$

FTC - part II

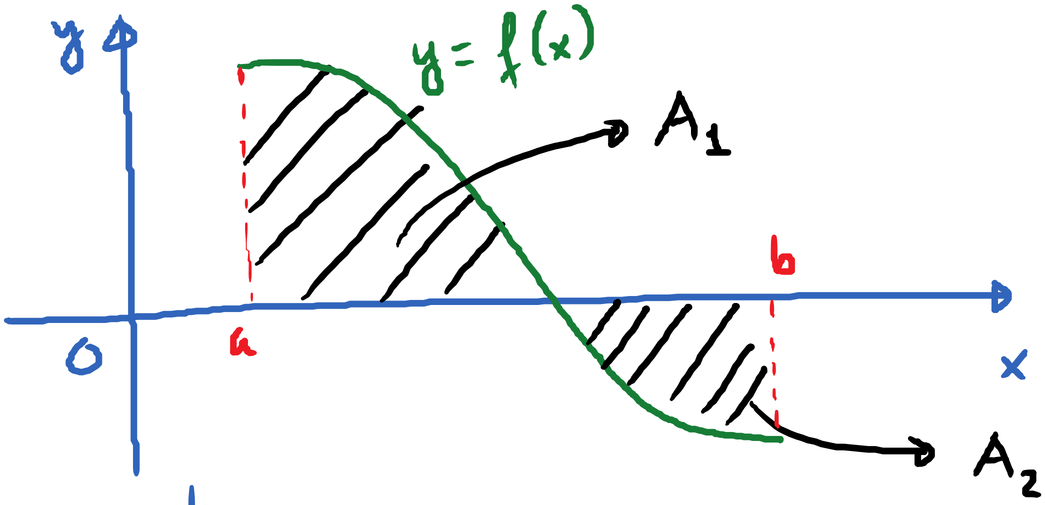
$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$

Areas between curves.



$$\underbrace{\int_a^b}_{\text{Infinite Sum}} \overbrace{\underbrace{f(x)}_{\text{height}}\,\underbrace{dx}_{\text{width}}}^{\text{area of rectangle}} = \text{Area under the curve } y = f(x) \text{ from } x = a \text{ to } x = b$$



$$\int_a^b f(x)\,dx = A_1 - A_2$$ (= signed area between f(x) and x-axis.

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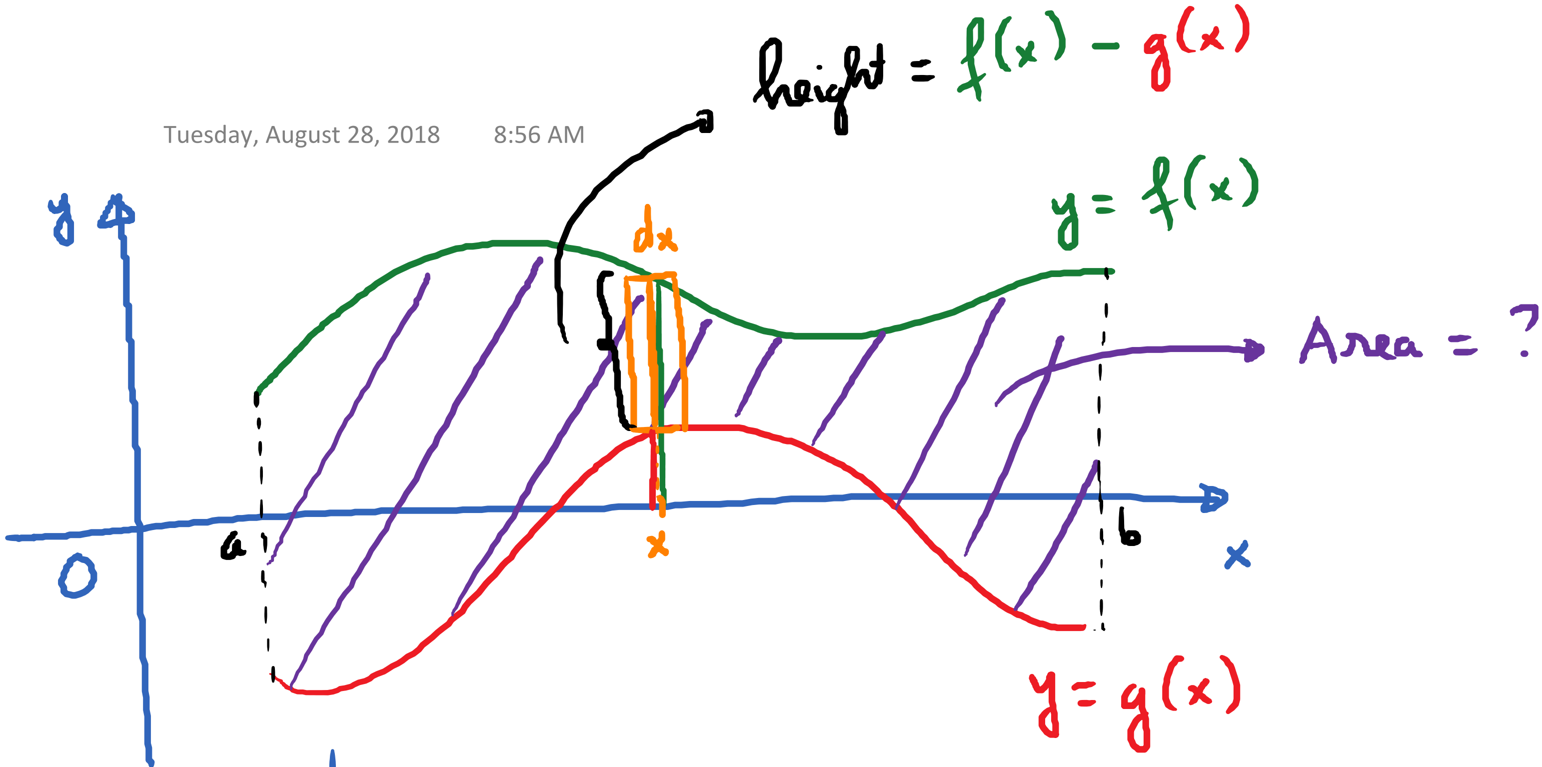

$$A = \int_a^b \underbrace{\underbrace{[f(x) - g(x)]}_{\text{height}} \; \underbrace{dx}_{\text{width}}}_{\text{area of a small rectangle}}$$

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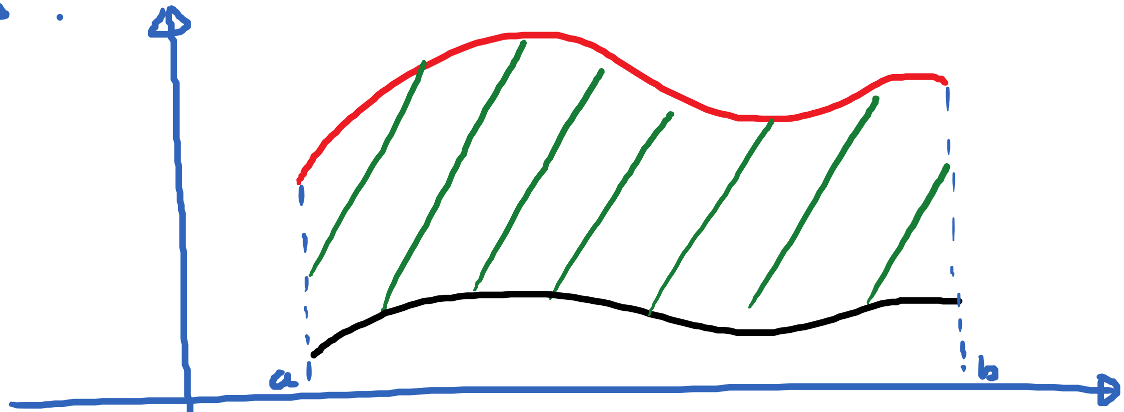
In general, Formula for the area of 2 non-intersecting curves:



$$\boxed{\text{Shaded Area} = \int_a^b (\text{top curve} - \text{bottom curve})\,dx}$$

What if we have intersecting curves?

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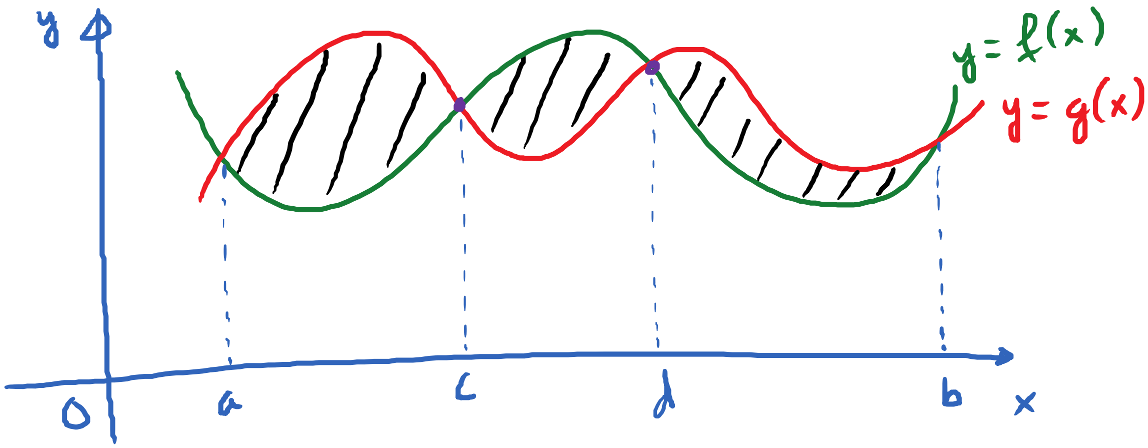

Shaded Area = ?

Step 1: Set $f(x) = g(x)$ to solve for the x-coordinates of the points of intersection.

Step 2:
$$\text{Area} = \int_a^c [g(x) - f(x)]\,dx + \int_c^d [f(x) - g(x)]\,dx + \int_d^b [g(x) - f(x)]\,dx$$