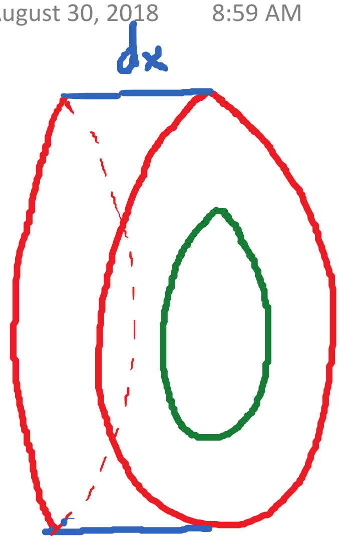
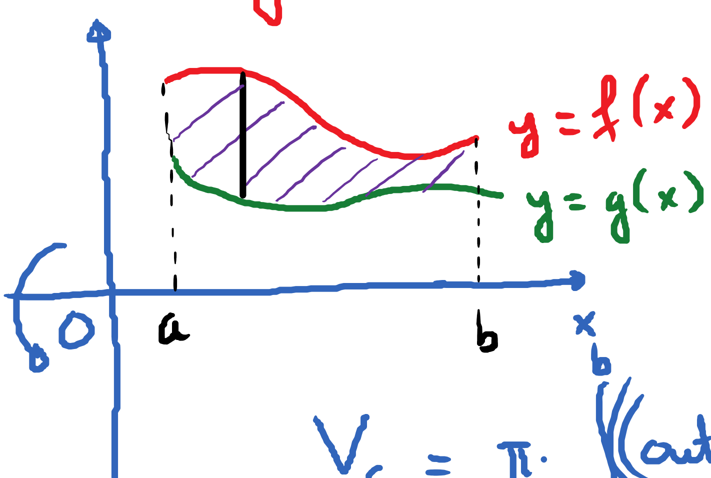


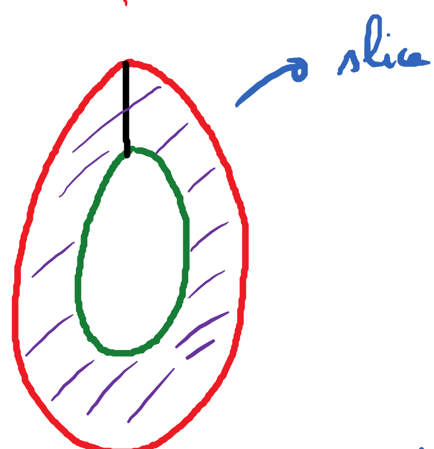
$$V_{\text{"thicken slice"}} = \left(\pi\cdot[x]^2 - \pi\cdot[x^2]^2\right)dx$$

$$V_{\text{Solid}} = \pi\int_0^1 \left(x^2 - x^4\right)dx = \pi\cdot\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \pi\cdot\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

Summary of Washer Method to find volumes.



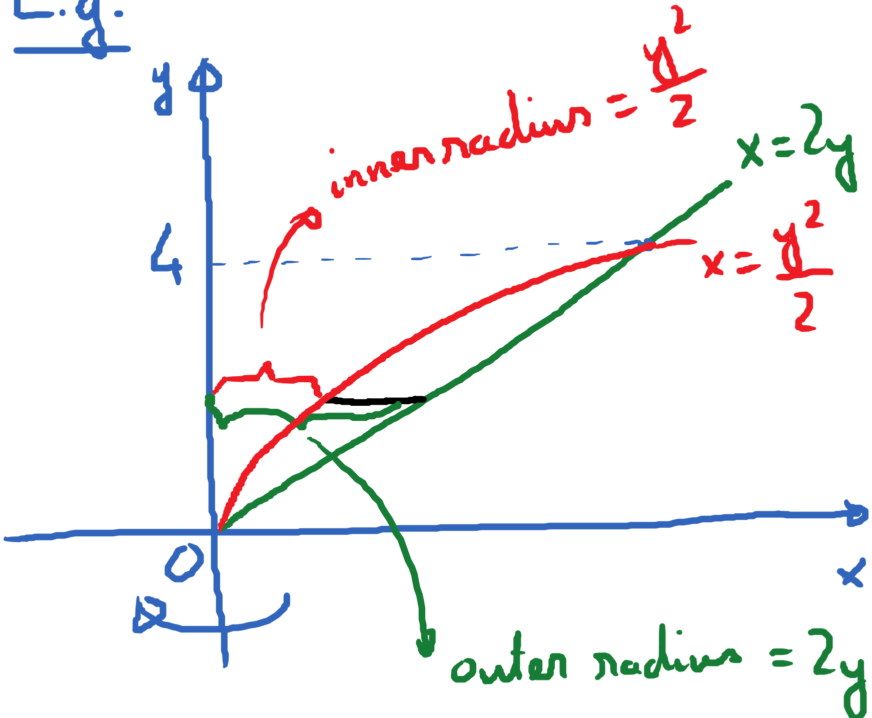


$$V_S = \pi\cdot\int_a^b\Big(\underbrace{(\text{outer radius})}_{f(x)}^2 - \underbrace{(\text{inner radius})}_{g(x)}^2\Big)dx$$

$$V_S = \pi \cdot \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

E.g.

Set $2y = \frac{y^2}{2} \to y^2 = 4y$
$\to y = 4; 0$



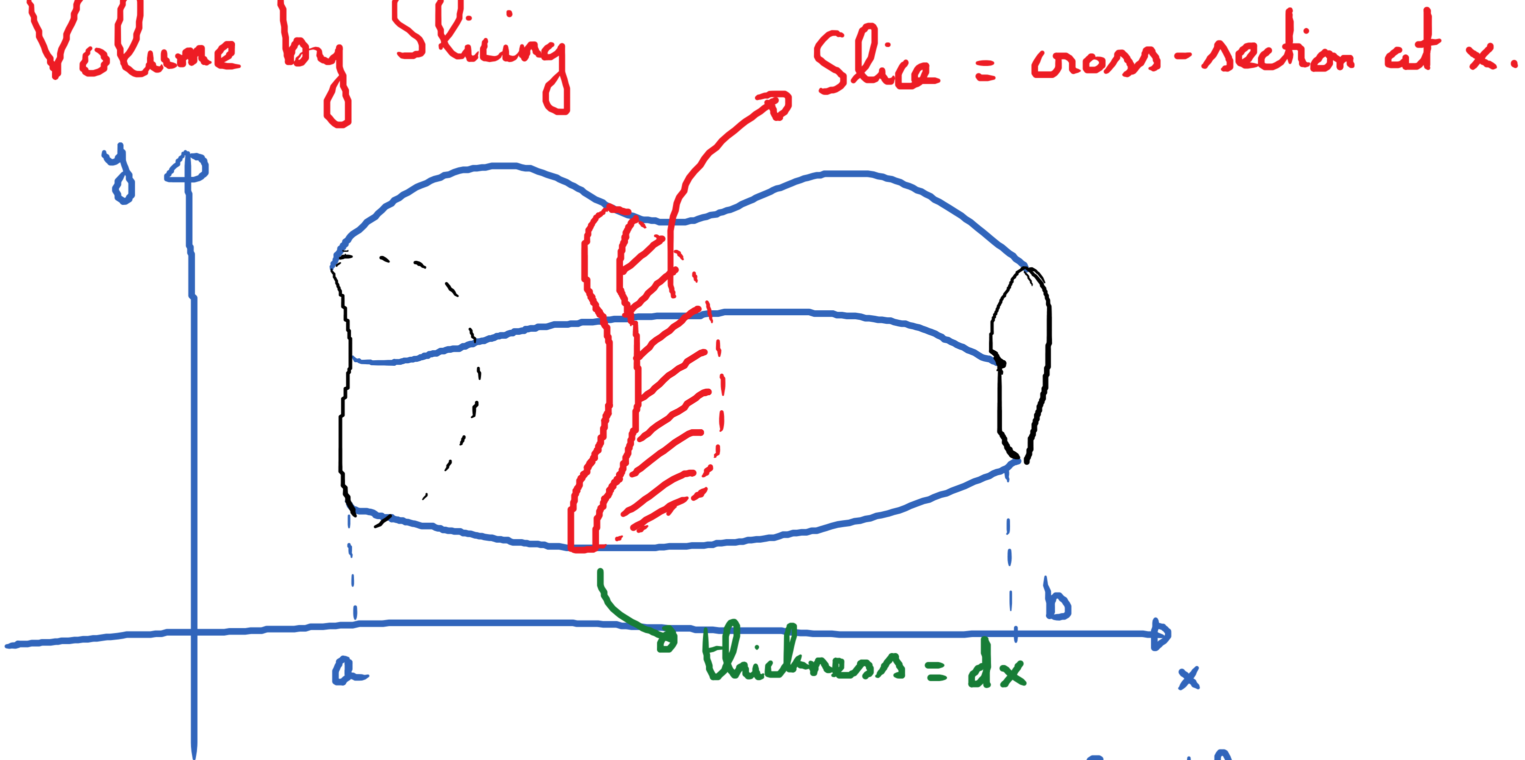
slice = washer

outer radius

$$V = \pi \int_0^4 \left((\text{outer radius})^2 - (\text{inner radius})^2 \right) dy$$

$$= \pi \cdot \int_0^4 \left(4y^2 - \frac{y^4}{4} \right) dy = \cdots = \boxed{\frac{512\pi}{15}}$$

Volume by Slicing



Key: If you can find the formula for the cross-sectional area at x, then you can find the volume.

Say, the cross-sectional area is given by a function of x, cross-sectional area = $A(x)$

$$V_{\text{"thickened" slice}} = A(x)\,dx$$

$$\rightarrow V_{object} = \int_a^b A(x)\,dx$$

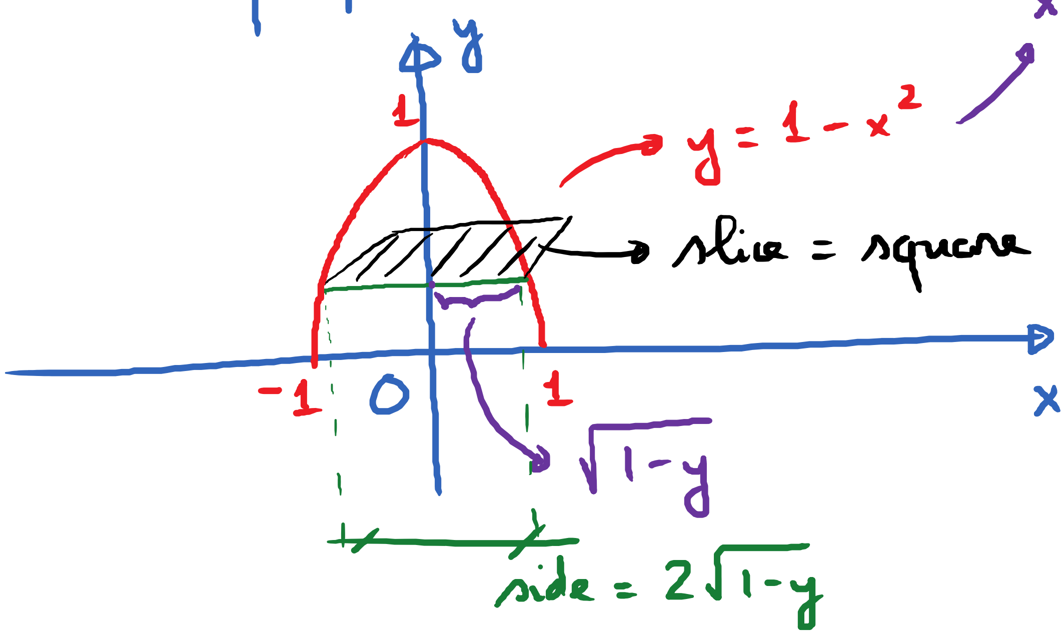
If slices are perpendicular to y-axis, then:

$$V_{object} = \int_a^b A(y)\,dy$$

E.g. #16. Base of object: region under parabola $y = 1 - x^2$ and above the x-axis.

Cross sections perpendicular to y-axis are squares

$x = \pm\sqrt{1-y}$



Area of slice = Area of a square = $(\text{side})^2$.

$= \left(2\sqrt{1-y}\right)^2$.

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left(2\sqrt{1-y}\right)^2 dy$$

$$= \int_0^1 4(1-y)\,dy = 4\cdot\left(y - \frac{y^2}{2}\right)\Bigg|_0^1$$

$$= \boxed{2}.$$