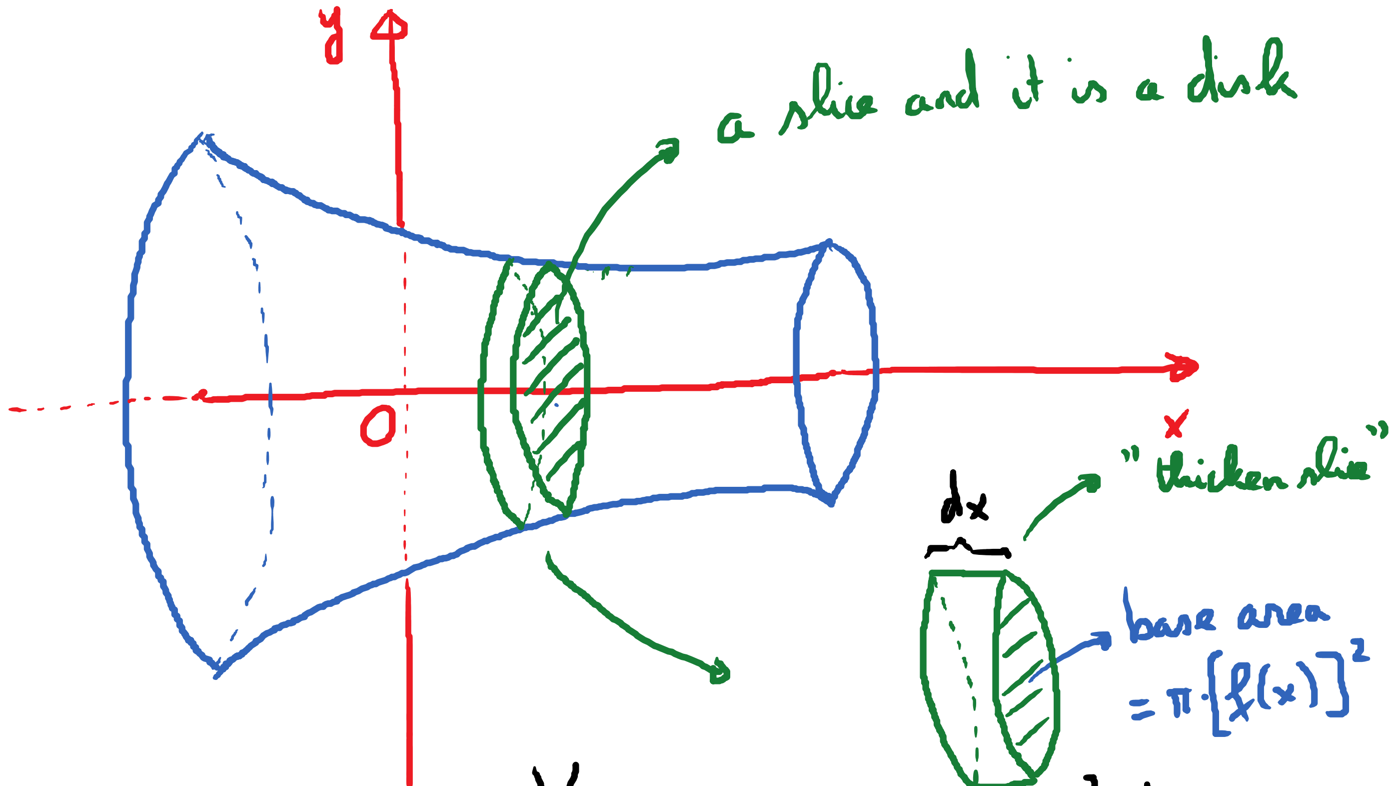
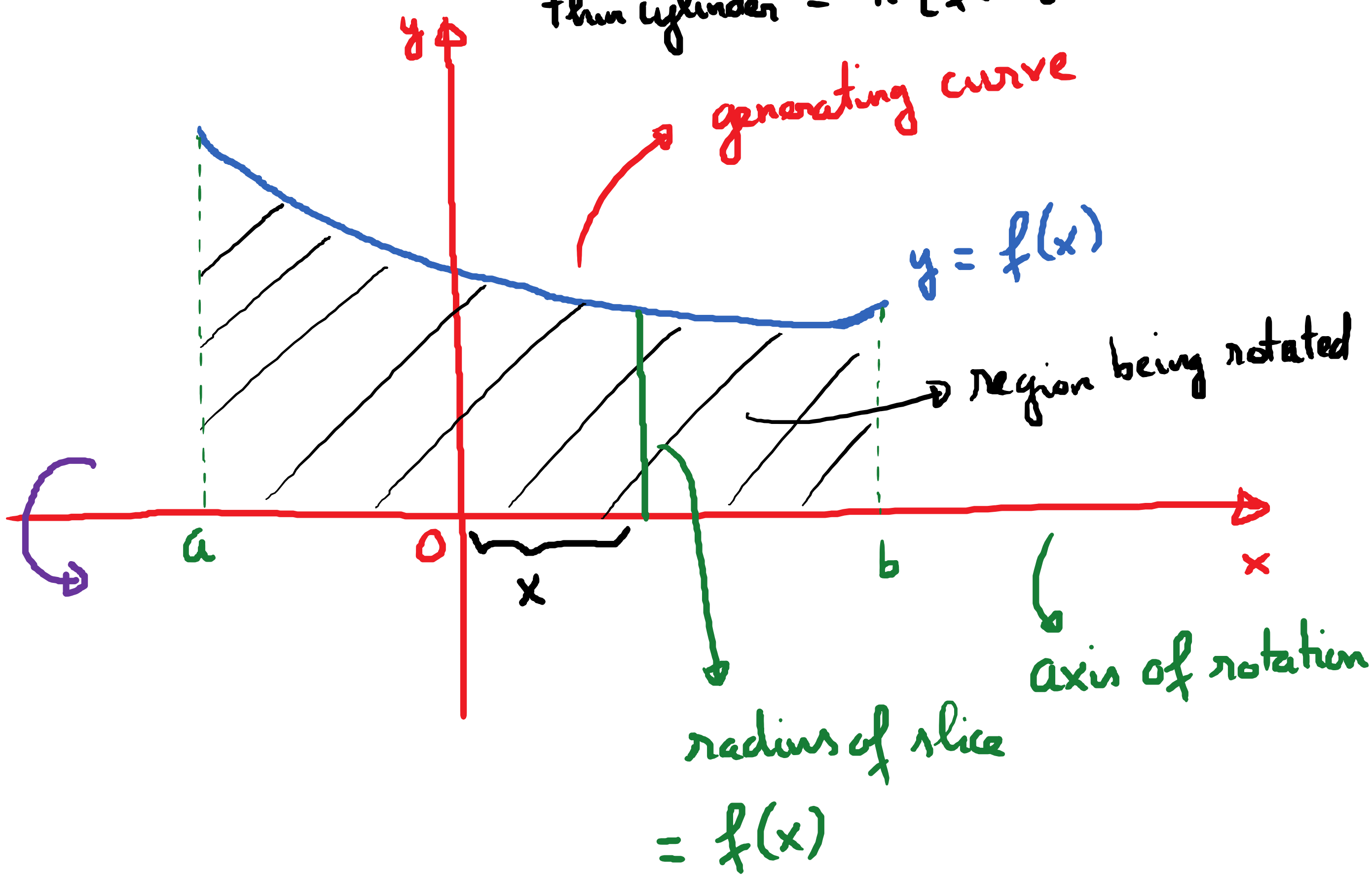
2.2. Volume by Slicing | Disk-Washer Method

Thursday, August 30, 2018 7:58 AM



$$V_{\text{thin cylinder}} = \pi \cdot [f(x)]^2 \cdot dx$$



Volume of the solid obtained by rotating the region bounded by $y = f(x)$, $a \le x \le b$ and the x-axis about the x-axis:

$$\boxed{V_{solid} = \pi \int_a^b [f(x)]^2 \, dx}$$

(this is called the disk method b/c a slice is a disk)

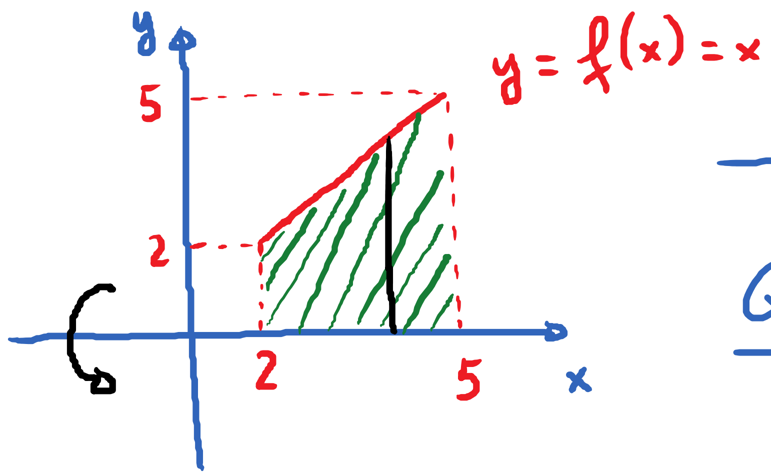
Why is this formula true?

Area of a slice (at x) $= \pi \cdot (\text{radius})^2 = \pi \cdot (f(x))^2$

$\longrightarrow$ thicken slice $\rightarrow$ $V_{\text{"thicken slice"}} = \underbrace{\pi \cdot [f(x)]^2}_{\text{base area}} \cdot \underbrace{dx}_{\text{height}}$

$$V_{solid} = \sum V_{\text{"thicken slice"}} = \int_a^b \pi [f(x)]^2 \cdot dx$$

Ex. $y = f(x) = x$. Rotate the shaded region by the x-axis.



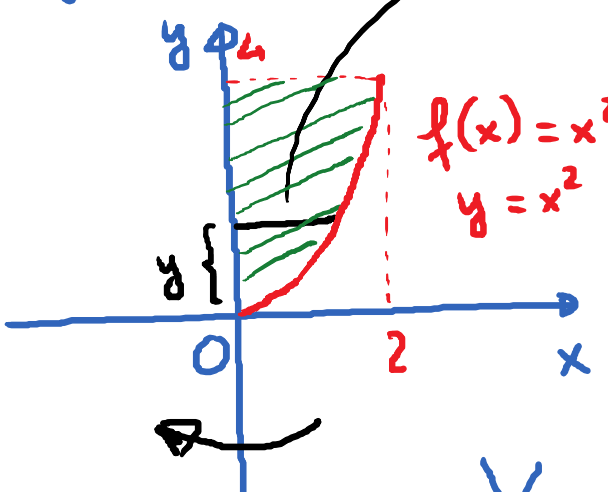
$\longrightarrow$ Obtain a solid S.

Q: $V_S = ?$

$$V_S = \pi \int_2^5 x^2\,dx = \pi \cdot \frac{x^3}{3}\Bigg|_2^5 = \pi \cdot \left(\frac{(5)^3}{3} - \frac{(2)^3}{3}\right)$$

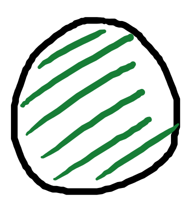
$$= \frac{117\pi}{3} = \boxed{39\pi}$$

E.g. $f(x) = x^2$ radius $\overset{?}{=} \sqrt{y}$



$f(x) = x^2$

$y = x^2 \longrightarrow x = \sqrt{y}$



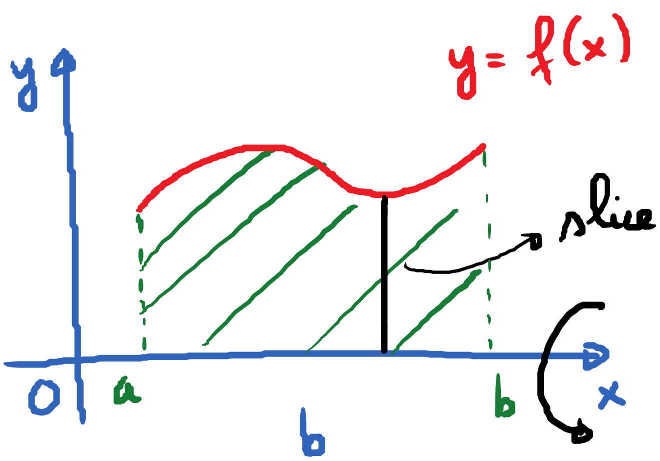
Area of disk $= \pi \cdot (\sqrt{y})^2$

$V_{\text{thicken slice}} = \pi (\sqrt{y})^2 \cdot dy$

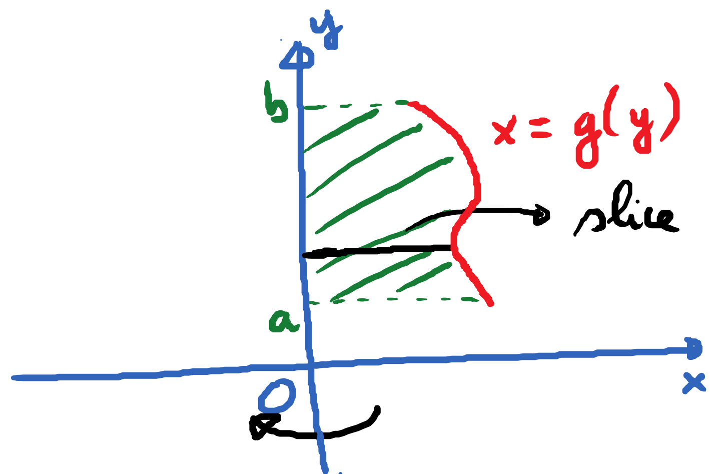
$\longrightarrow V_{\text{solid obtained}} = \pi \int_0^4 (\sqrt{y})^2\,dy$

$$V = \pi \cdot \int_0^4 y\,dy = \pi \cdot \left(\frac{y^2}{2}\right)\Big|_0^4 = \pi \cdot (8) = \boxed{8\pi}$$

Summary: Finding Volume by the disk method.



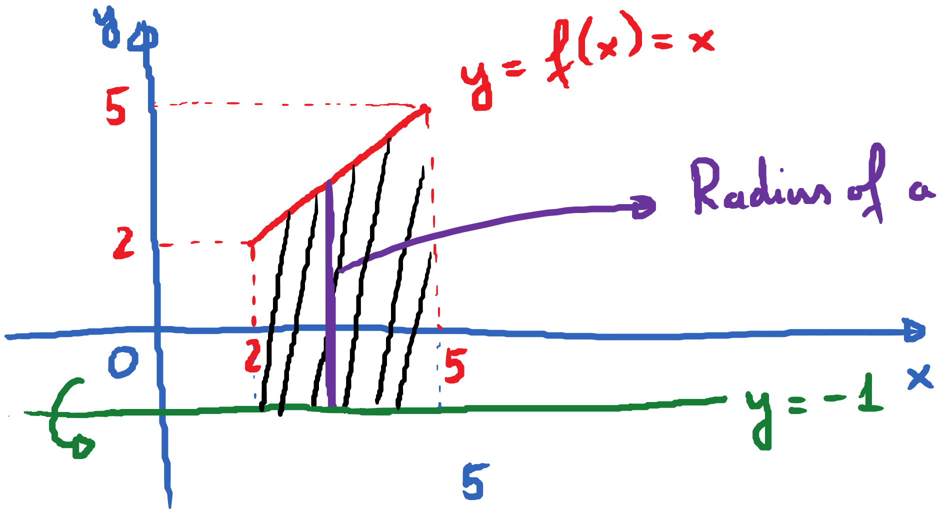
$$V_{\text{Solid}} = \pi \cdot \int_a^b [f(x)]^2\,dx$$



$$V_{\text{solid}} = \pi \cdot \int_a^b [g(y)]^2\,dy$$

Rotating about an axis different from x-axis or y-axis

E.g. $y = f(x) = x$; $2 \leq x \leq 5$.

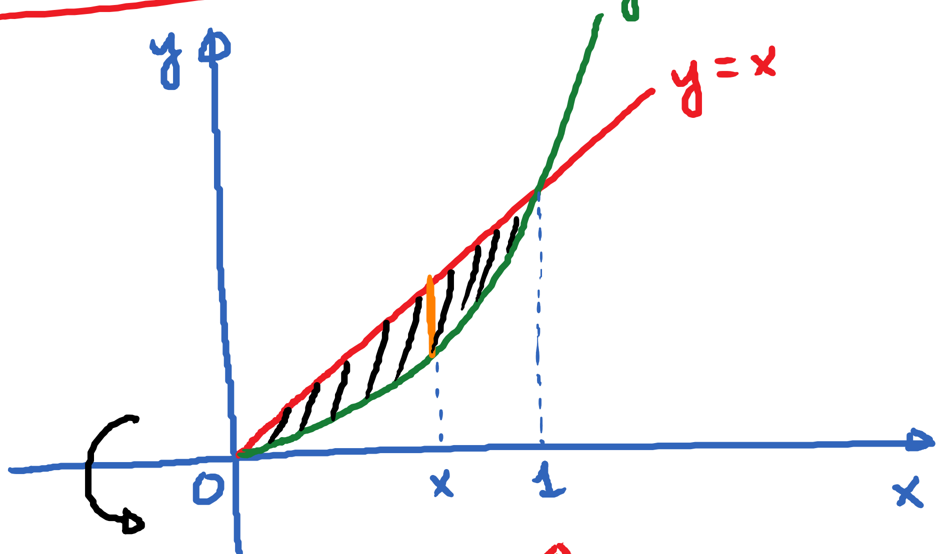


Radius of a slice at $x = f(x) + 1$

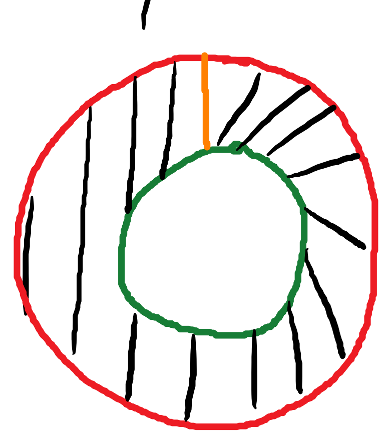
$= x + 1.$

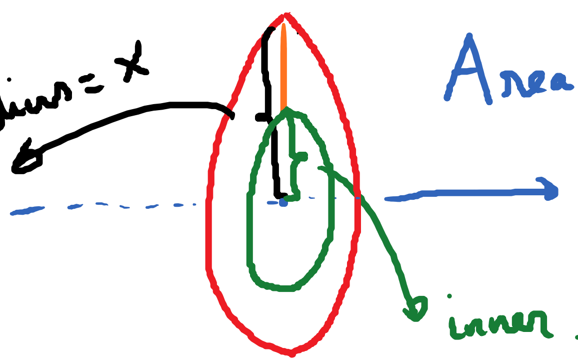
$$V = \pi \cdot \int_2^5 (x+1)^2 \, dx$$

Washer Method:



slice = washer





outer radius = x

inner radius = x^2

$$\text{Area of slice} = \pi \cdot (\underbrace{\text{outer radius}}_{x})^2 - \pi \cdot (\underbrace{\text{inner radius}}_{x^2})^2$$