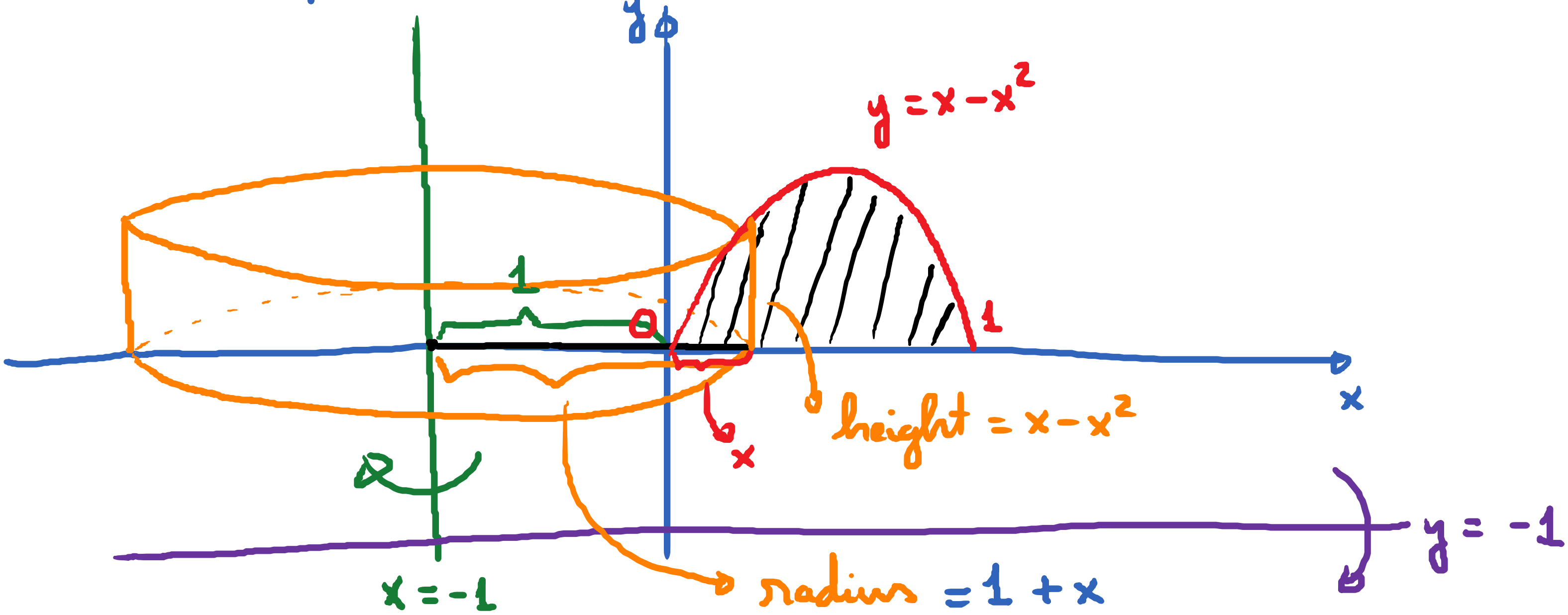
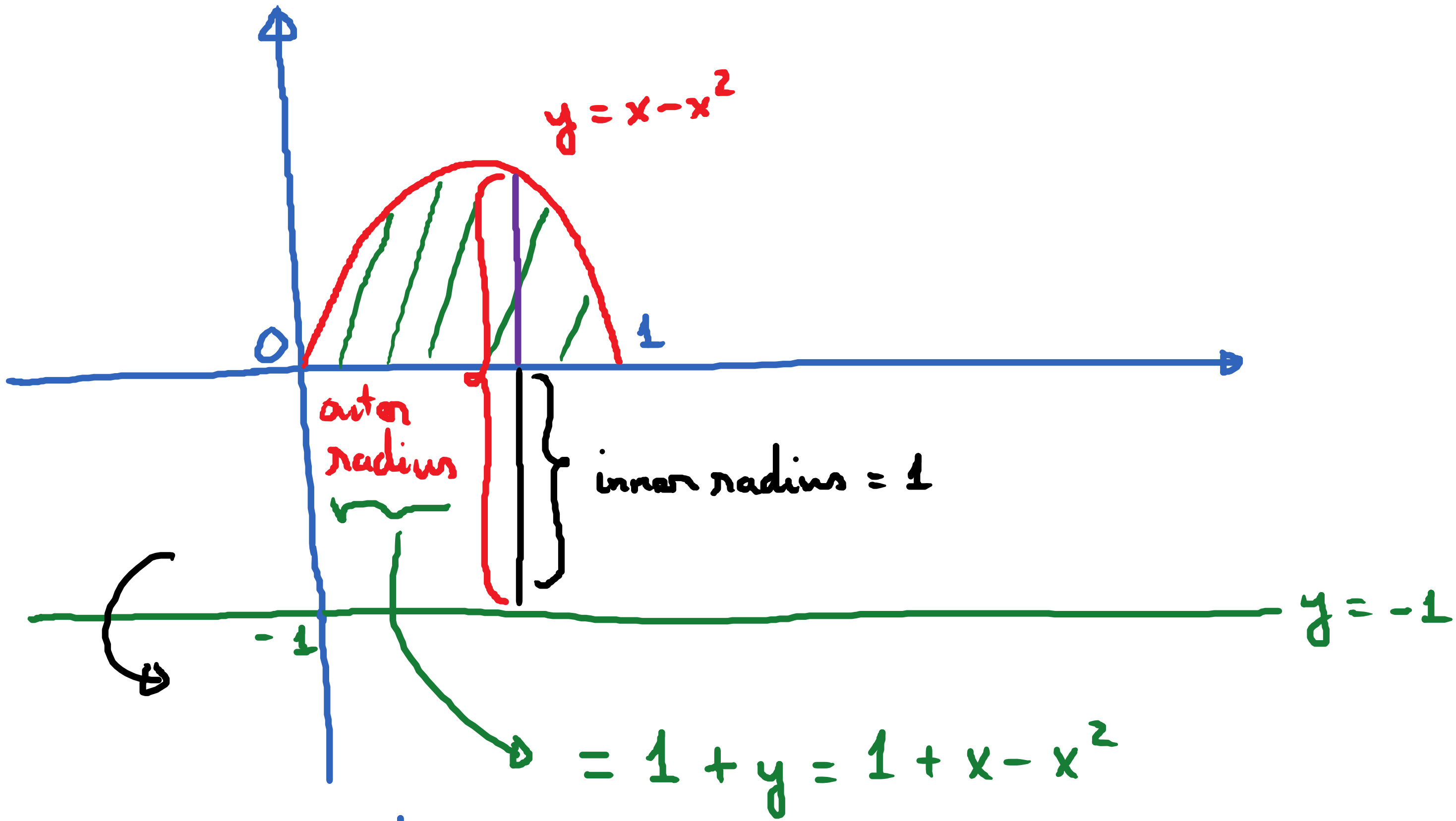
More questions:

* Rotate shaded region about $\boxed{x=-1}$.
  Set up the integral to find the volume.
* Rotate shaded region about $\boxed{y=-1}$.
  Set up the integral to find the volume



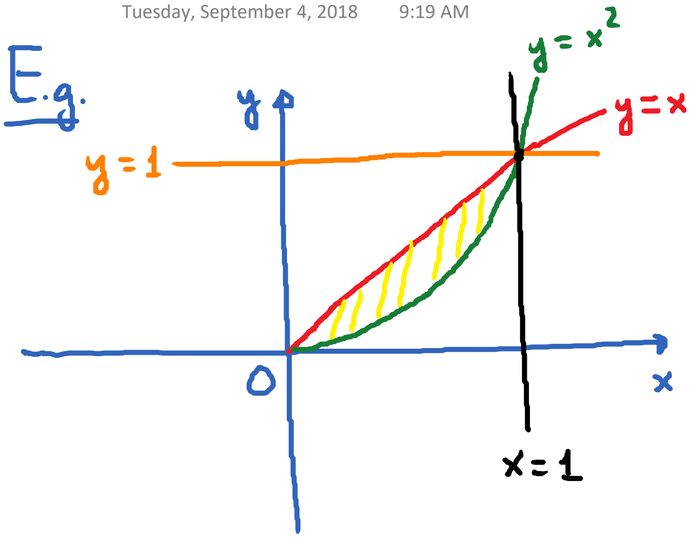
$\boxed{x=-1}$ $$V = 2\pi \cdot \int_0^1 \underbrace{(x+1)}_{\text{radius}} \cdot \underbrace{(x-x^2)}_{\text{height}} \, \underbrace{dx}_{\text{thickness}}$$



$$V_S = \pi \cdot \int_0^1 \left[\left(1 + x - x^2\right)^2 - 1 \right] dx$$

What if we have a region bounded by 2 functions?

E.g.



Q: Set up the integral to find the volume of the solid obtained by:

① Rotating the shaded region about $\boxed{x = 1}$ by shell method and by washer method.

② Rotating the shaded region about $\boxed{y = 1}$ by shell method and by washer method.

Sol: ① Shell: $V = 2\pi \cdot \int_0^1 (1-x)(x-x^2)\,dx$

Washer: $V = \pi \int_0^1 \left[(1-y)^2 - (1-\sqrt{y})^2\right] dy$

② Shell:

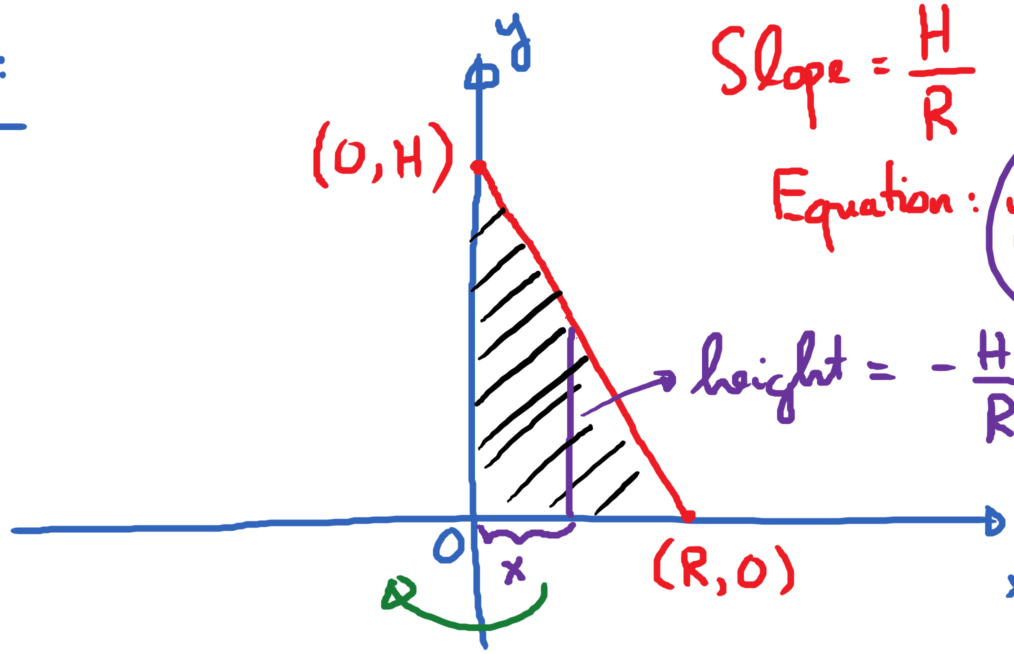
$$V = 2\pi \cdot \int_0^1 (1-y)\cdot(\sqrt{y} - y)\,dy$$

Washer

$$V = \pi \cdot \int_0^1 \left[(1-x^2)^2 - (1-x)^2\right] dx$$

Volumes of Familiar Shapes

Cone:



Slope $= \frac{H}{R}$

Equation: $y = -\frac{H}{R}x + H$

height $= -\frac{H}{R}x + H$

$$V = 2\pi \cdot \int_0^R x \cdot \left(-\frac{H}{R}x + H\right) dx$$

$$= 2\pi \cdot \int_0^R \left(-\frac{H}{R}x^2 + Hx\right) dx$$

$$= 2\pi \cdot \left(-\frac{H}{R} \cdot \frac{x^3}{3} + H \cdot \frac{x^2}{2}\right)\Bigg|_0^R$$

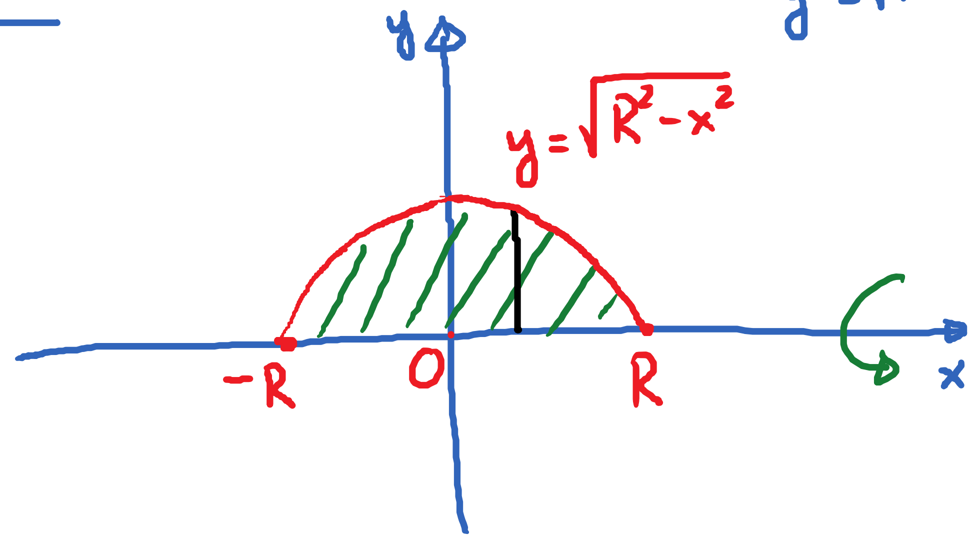
$$= 2\pi \cdot \left(-\frac{H}{\cancel{R}} \cdot \frac{R^{\cancel{3}\,2}}{3} + H \cdot \frac{R^2}{2}\right)$$

$$= 2\pi \cdot \left(-\frac{HR^2}{3} + \frac{HR^2}{2}\right) = 2\pi \cdot \frac{HR^2}{6}$$

$$= \boxed{\frac{1}{3}\pi R^2 H}$$

Sphere:

$$x^2 + y^2 = R^2$$
$$y = \sqrt{R^2 - x^2}$$



By disk method,

$$V = \pi \cdot \int_{-R}^{R} \Big(\underbrace{\sqrt{R^2 - x^2}}_{\text{radius}}\Big)^2 \, dx$$

$$= \pi \cdot \int_{-R}^{R} (R^2 - x^2)\, dx = \pi \cdot \left(R^2 x - \frac{x^3}{3}\right)\Bigg|_{-R}^{R}$$

$$= \pi \left[\left(R^3 - \frac{R^3}{3}\right) - \left(-R^3 + \frac{R^3}{3}\right)\right] = \pi \cdot \left(\frac{2R^3}{3} + \frac{2R^3}{3}\right)$$

$$= \boxed{\frac{4\pi R^3}{3}}$$