2.3. Volumes by Cylindrical Shell

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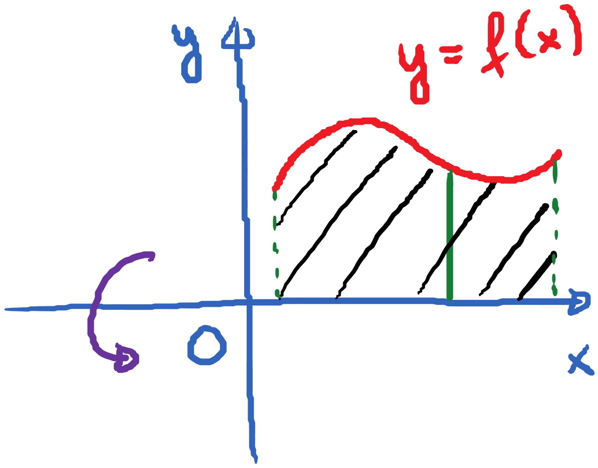
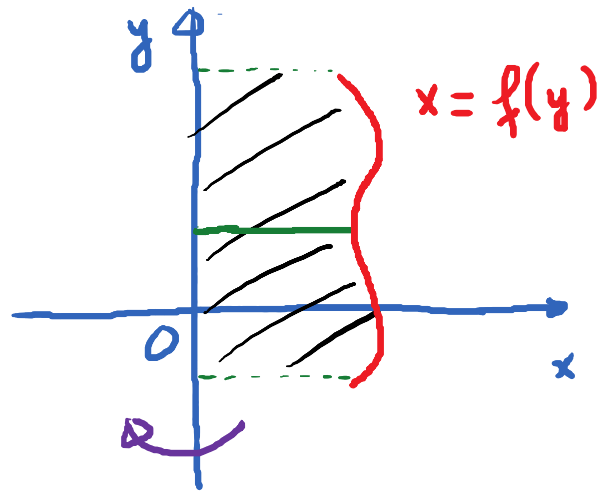
Recall: Last time Disk and Washer Method

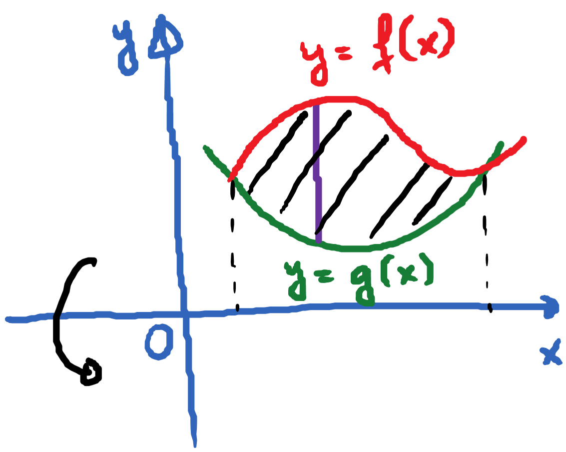


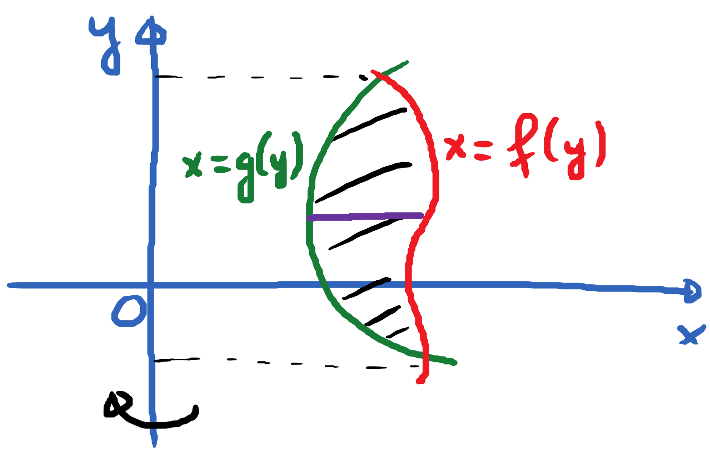


Cross-sections are disks

given by f

$$\text{Volume} = \pi \int (\boxed{\text{radius}})^2$$



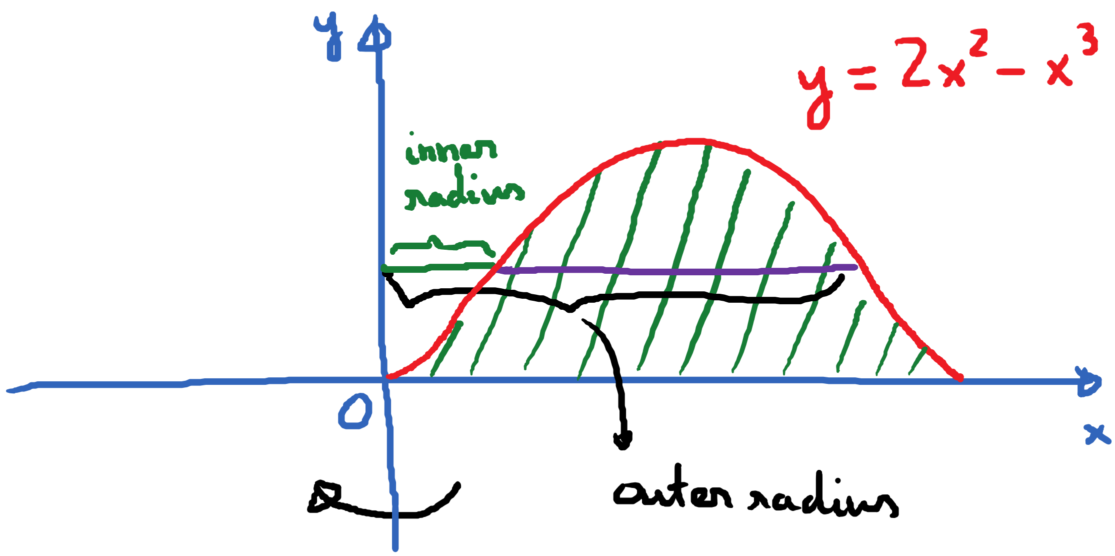


Cross-sections are washers

$$\text{Volume} = \pi \cdot \int \Big(\underbrace{\text{outer radius}}_{f}\Big)^2 - \Big(\underbrace{\text{inner radius}}_{g}\Big)^2$$

In both methods, cross-sections are perpendicular to the axis of rotation.

E.g.



Rotate the region bounded by the graph of $y = 2x^2 - x^3$ and the x-axis about the y-axis.

Q: Find the volume of the solid obtained.

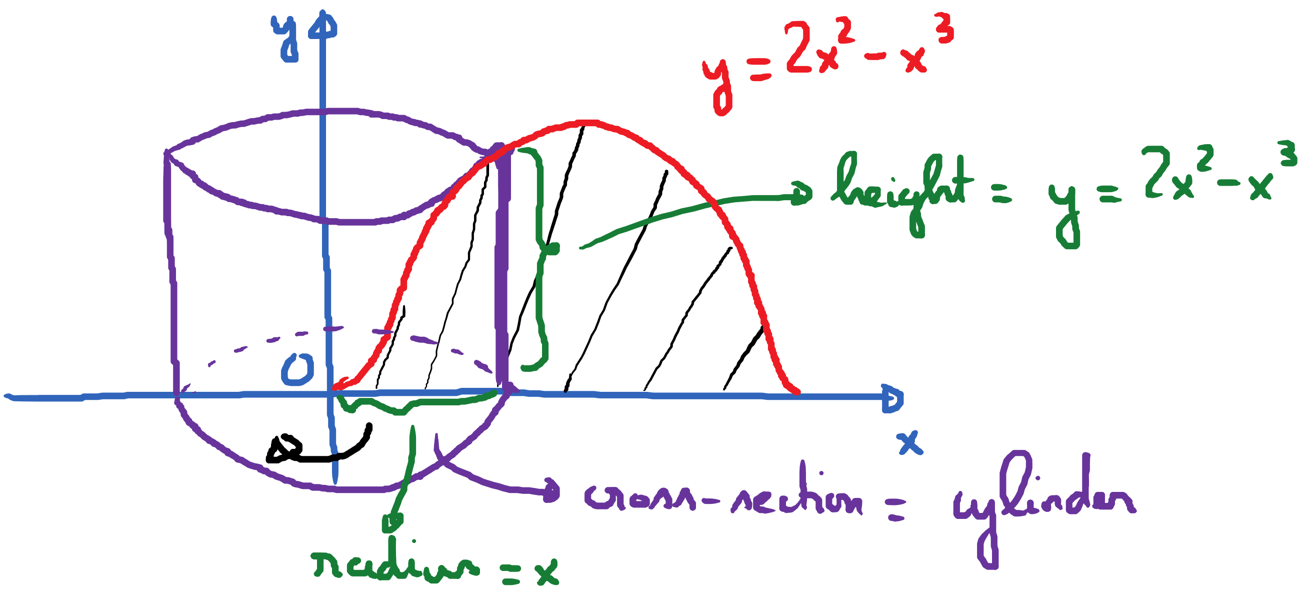
If we use washer method, then

$$V = \pi \int (\text{outer radius})^2 - (\text{inner radius})^2$$

To find inner radius and outer radius formula, we have to solve for x in terms of y

⟶ It does not work!

⟶ Shell method: Slice the region parallell to the axis of rotation

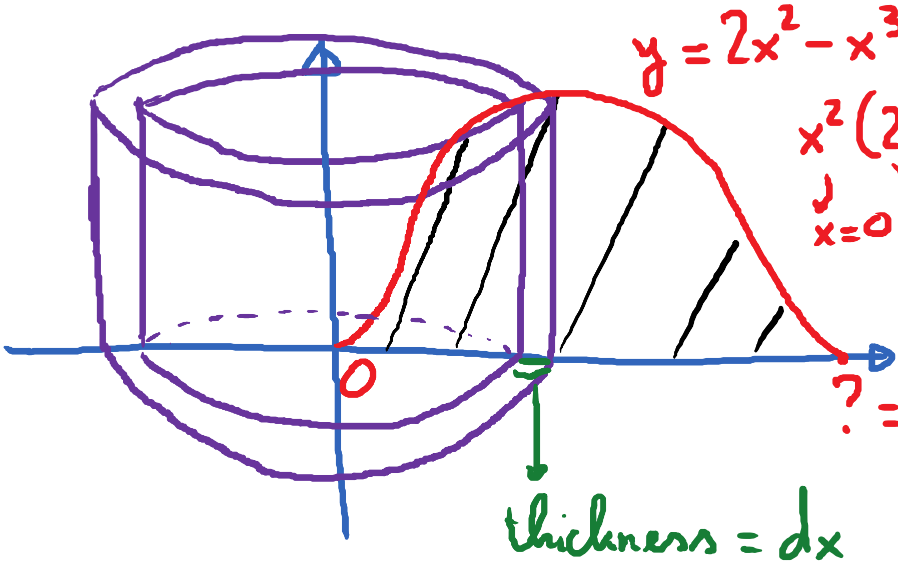


Cross-sectional area $= 2\pi \cdot \underbrace{(\text{radius})}_{x} \cdot \underbrace{(\text{height})}_{f(x)}$

$y = 2x^2 - x^3 = 0$

$x^2(2-x) = 0$

$x = 0 \qquad x = 2$

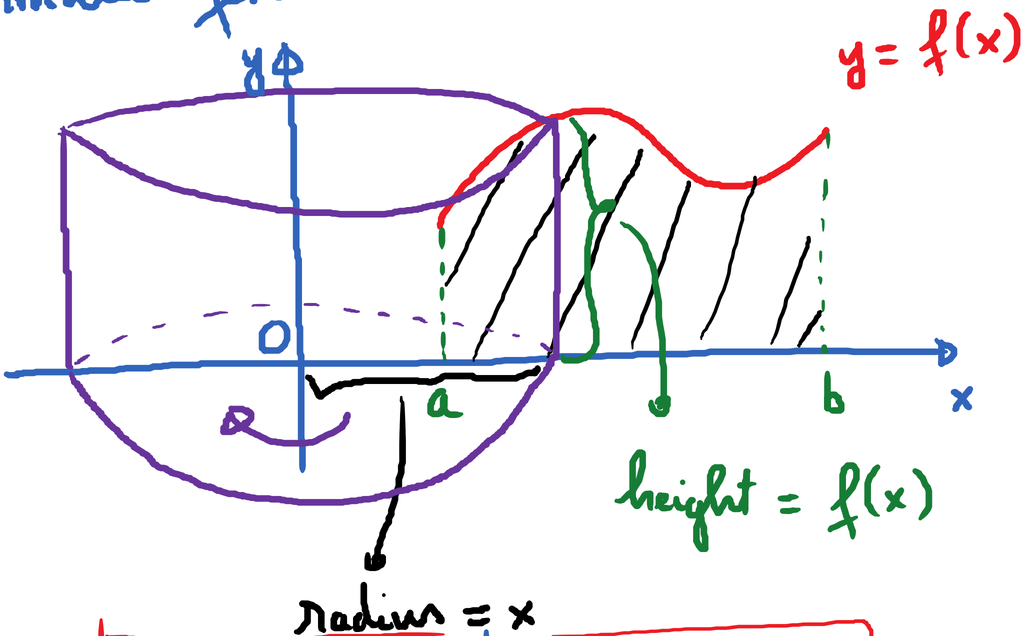


Volume of Solid $= 2\pi \cdot \int_0^2 x \cdot (2x^2 - x^3) \cdot dx$

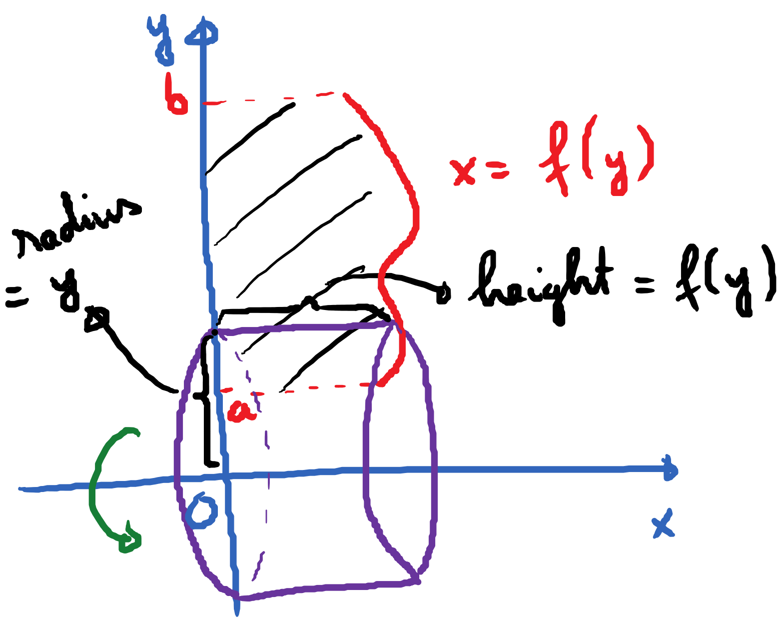
$$= 2\pi \cdot \int_0^2 (2x^3 - x^4)\, dx = 2\pi \cdot \left(\frac{x^4}{2} - \frac{x^5}{5}\right)\Bigg|_0^2$$

$$= 2\pi\left(\frac{16}{2} - \frac{32}{5}\right) = \boxed{\frac{16\pi}{5}}$$

Formula for Shell Method



$$\boxed{V_S = 2\pi \int_a^b x \cdot f(x) \cdot dx}$$



$$\boxed{V_S = 2\pi \cdot \int_a^b y \cdot f(y) dy}$$

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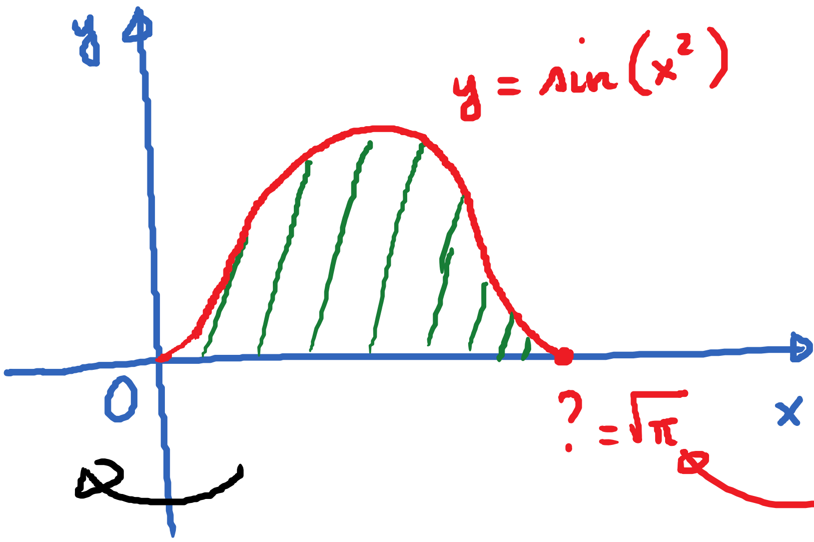
Ex.



Rotate the shaded region about the y-axis.

Q: Find the volume of the solid obtained?

To find ?, set $\sin(x^2) = 0$

$x^2 = 0$ or $x^2 = \pi$

$x = 0$ $\quad x = \sqrt{\pi}$

$$V_S = 2\pi \cdot \int_0^{\sqrt{\pi}} x \sin(\boxed{x^2})\, dx$$

u

Let $u = x^2$. Then $du = 2x\,dx$. Then

$$V_S = \pi \int_0^{\pi} \sin(u)\, du = \pi \cdot \left(-\cos(u)\right)\Big|_0^{\pi}$$

$$= \pi \cdot \Big(\underbrace{-\cos(\pi)}_{1} + \underbrace{\cos(0)}_{1}\Big) = \boxed{2\pi}$$

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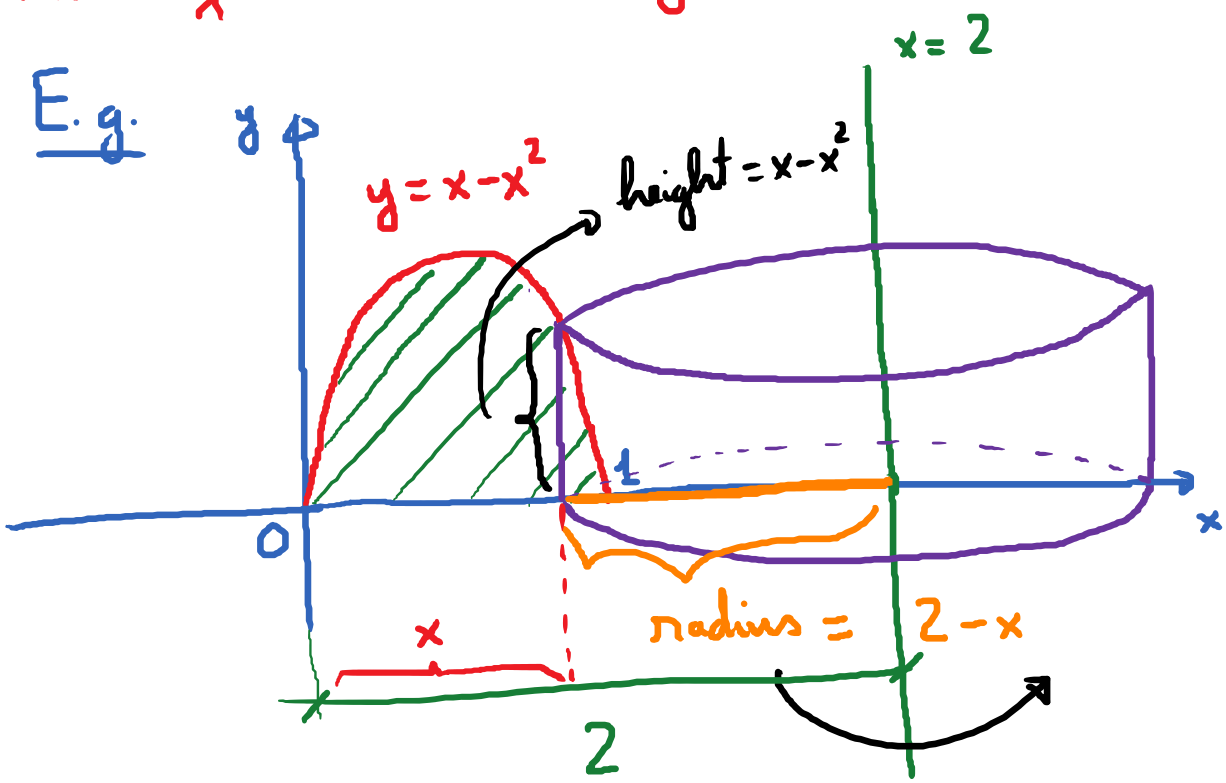
Axis of rotation is any vertical or horizontal line.

E.g.



Using shell method to find the volume of the solid obtained by rotating the shaded region about $x = 2$

$$V = 2\pi \int_0^1 \underbrace{(2-x)}_{\text{radius}} \cdot \underbrace{(x - x^2)}_{\text{height}} \cdot \underbrace{dx}_{\text{thickness}}$$

⟶ Foil it out & integrate.