$$ds = \sqrt{1 + \left(\frac{5}{2\sqrt{1+5x}}\right)^2}\, dx$$

$$= \sqrt{1 + \frac{25}{4(1+5x)}}\, dx$$

$$ds = \sqrt{\frac{4(1+5x)+25}{4(1+5x)}}\, dx = \sqrt{\frac{20x+29}{4(1+5x)}}\, dx$$

Step 2:

$$S = \cancel{2}\pi \cdot \int_1^7 \cancel{\sqrt{1+5x}} \cdot \frac{\sqrt{20x+29}}{\cancel{2}\cancel{\sqrt{1+5x}}}\, dx$$

$$S = \pi \cdot \int_{1}^{7} \sqrt{20x+29}\, dx$$

Let $u = 20x + 29$. Then $du = 20\, dx$.

So, $dx = \dfrac{du}{20}$.

$$S = \pi \cdot \int_{49}^{169} \sqrt{u} \cdot \frac{du}{20} = \frac{\pi}{20} \int_{49}^{169} u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{20} \cdot \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_{49}^{169} = \frac{\pi}{20} \cdot \frac{2}{3} \cdot \left((169)^{\frac{3}{2}} - (49)^{\frac{3}{2}} \right)$$

$$= \frac{\pi}{30} \cdot \left((13)^3 - (7)^3 \right) = \frac{1854\pi}{30} = \boxed{\frac{309\pi}{5}}$$

* Do this w.r.t. y.

1st Step: Get x in terms of y

$$y = \sqrt{1+5x} \rightarrow y^2 = 1+5x$$

$$\rightarrow x = \frac{y^2-1}{5} \longrightarrow g(y)$$

Bounds for y: Plug $x=1$: $y=\sqrt{6}$

$x=7$: $y = \sqrt{36} = 6$

2nd Step: $ds = \sqrt{1+[g'(y)]^2}\, dy$

$$g'(y) = \frac{2y}{5}.$$

$$ds = \sqrt{1+\left(\frac{2y}{5}\right)^2}\, dy = \sqrt{1+\frac{4y^2}{25}}\, dy$$

$$ds = \sqrt{\frac{25+4y^2}{25}}\, dy = \frac{1}{5}\sqrt{25+4y^2}\, dy.$$

Step 3:

$$S = 2\pi \cdot \int_{\sqrt{6}}^{6} y \cdot \frac{1}{5}\sqrt{25+4y^2}\, dy \quad ds$$

$$S = \frac{2\pi}{5}\cdot\frac{1}{8}\int_{\sqrt{6}}^{6} 8y\sqrt{\underbrace{25+4y^2}_{u}}\, dy \quad du$$

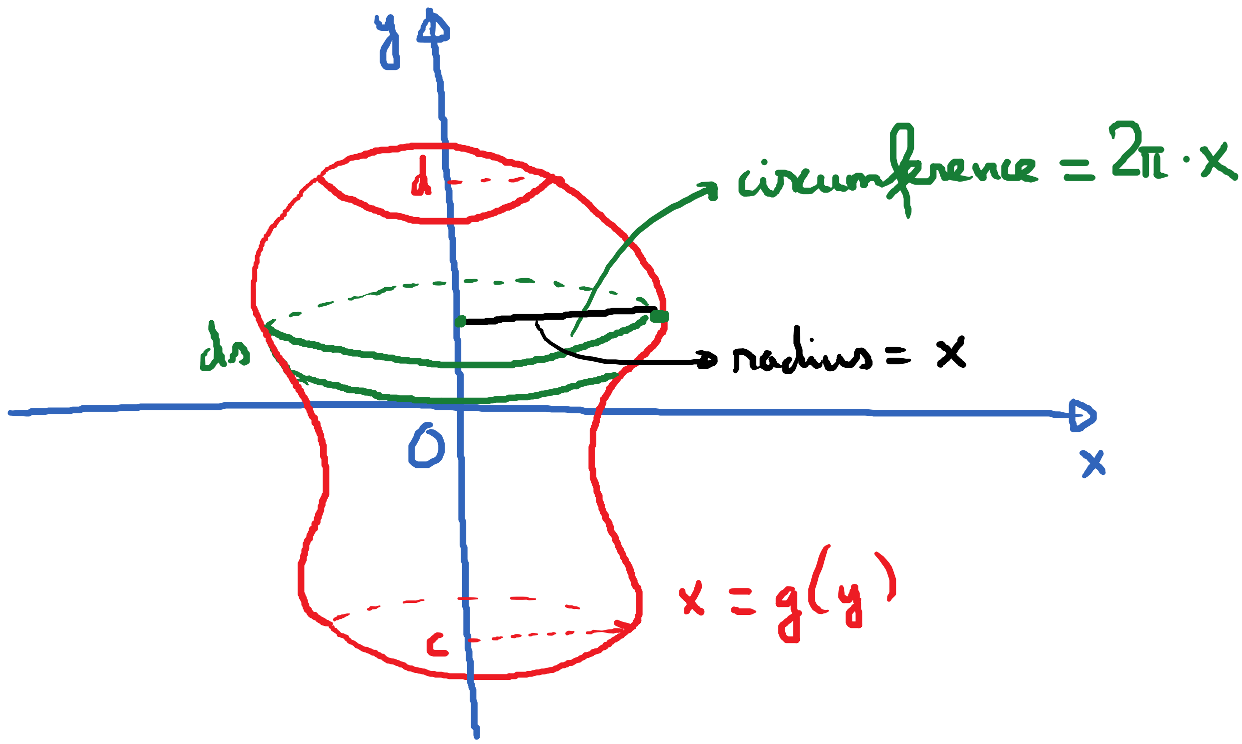
Let $u = 25+4y^2$. Then $du = 8y\,dy$.

$$S = \frac{\pi}{20}\int_{49}^{169} \sqrt{u}\, du = \frac{309\pi}{5}$$

Surface area

Thursday, September 6, 2018 9:56 AM

Note: What will the formula be if we rotate a curve about y-axis



area of a small piece $= 2\pi x \, ds$

Surface area $= 2\pi \int x \, ds$

2 ways to make this computable

I

$$S = 2\pi \int_c^d g(y)\sqrt{1 + [g'(y)]^2}\, dy$$

(Replace x by $g(y)$, find $g'(y)$ ....)

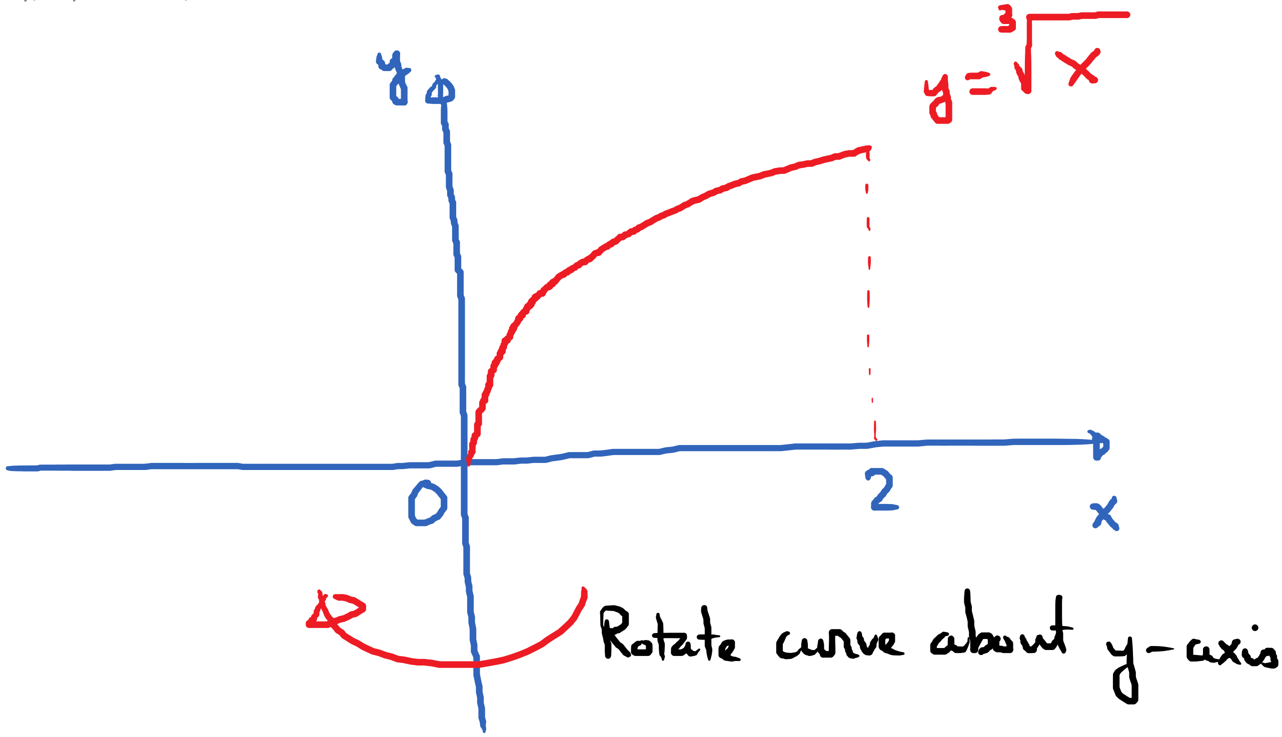
II

Solve for y in terms of x: $y = f(x)$

Find the bounds for x, say $a \le x \le b$.

$$S = 2\pi \cdot \int_a^b x \cdot \sqrt{1 + [f'(x)]^2}\, dx.$$

E.g.



Find surface area of resulting surface.

Let's say that our strategy is to integrate w.r.t. x

$$ds = \sqrt{1 + [f'(x)]^2}\, dx$$

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \rightarrow f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$ds = \sqrt{1 + \left(\frac{1}{3}x^{-\frac{2}{3}}\right)^2}\, dx = \sqrt{1 + \frac{1}{9}x^{-\frac{4}{3}}}\, dx$$

$$S = 2\pi \int_0^2 x \cdot \sqrt{1 + \frac{1}{9} x^{-4/3}} \, dx \longrightarrow \text{Messy!}$$

$\longrightarrow$ Do it w.r.t. y.