E.g. Find work done in moving an object along the x-axis from $x=1$ to $x=2$ with the variable force

$$f(x) = \frac{12}{x^2} \ (N)$$

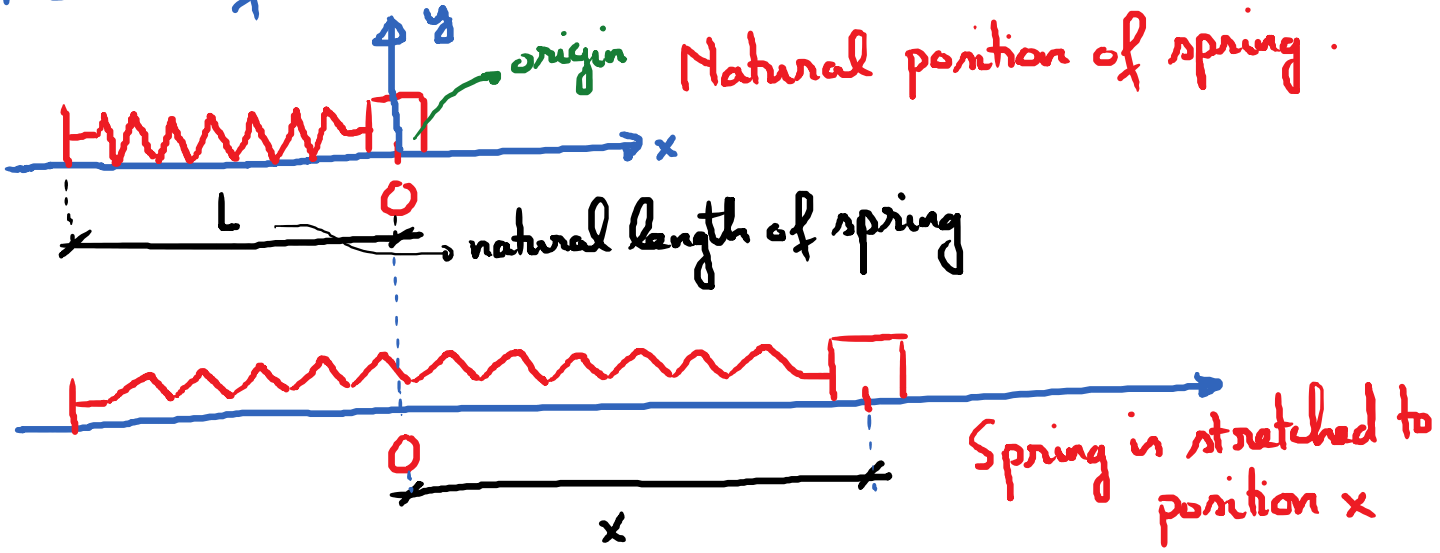
Sol:

$$W = \int_1^2 \frac{12}{x^2}\,dx = 12\int_1^2 x^{-2}\,dx = -\frac{12}{x}\Big|_1^2$$

$$= -6 + 12 = 6 \ (J)$$

* Work done in stretching / compressing a spring

Review of Hooke's Law in physics.



F: force required to stretch spring to position x and maintain it in this stretched position.

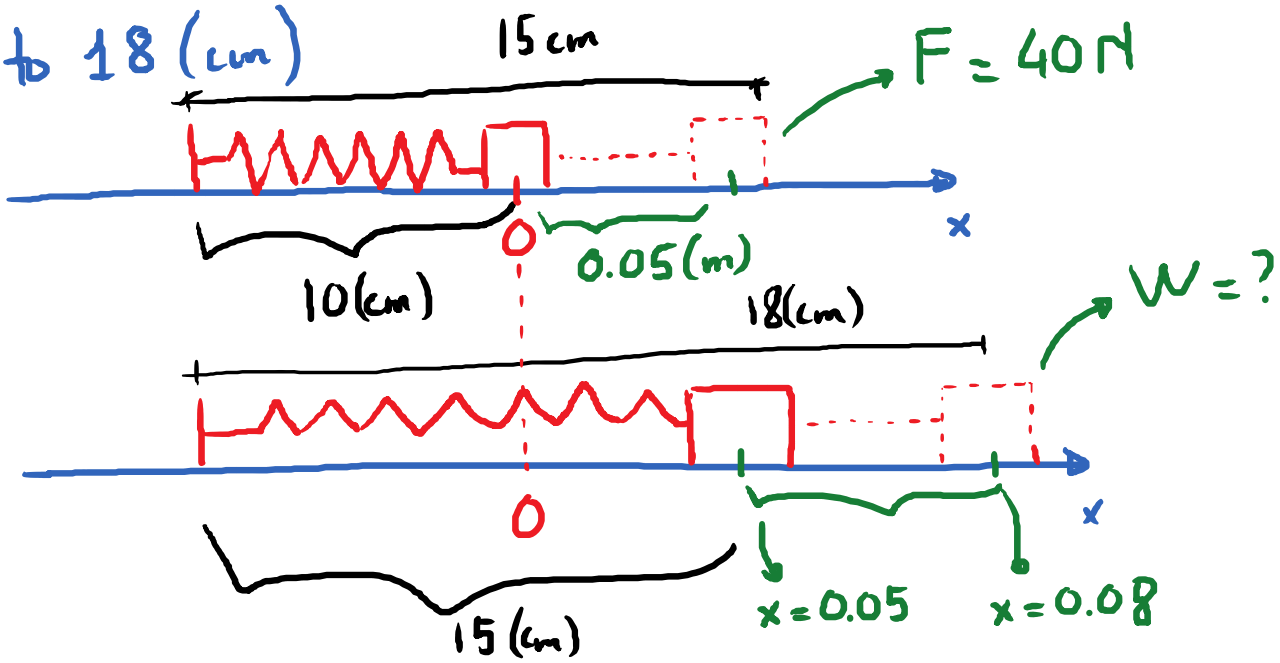
By Hooke's Law: $\boxed{F(x) = k \cdot x}$

(Here k is called the spring constant)

E.g. Spring has the natural length $L = 10(cm)$

* A force of 40 N is required to hold the spring when it is stretched from its natural length to a length of 15(cm).

* Q: Find work done in stretching the spring from 15(cm) to 18 (cm)



$F = kx$. When $x = 0.05$, $F = 40$.

So, $k = \frac{F}{x} = \frac{40}{0.05} = 800$

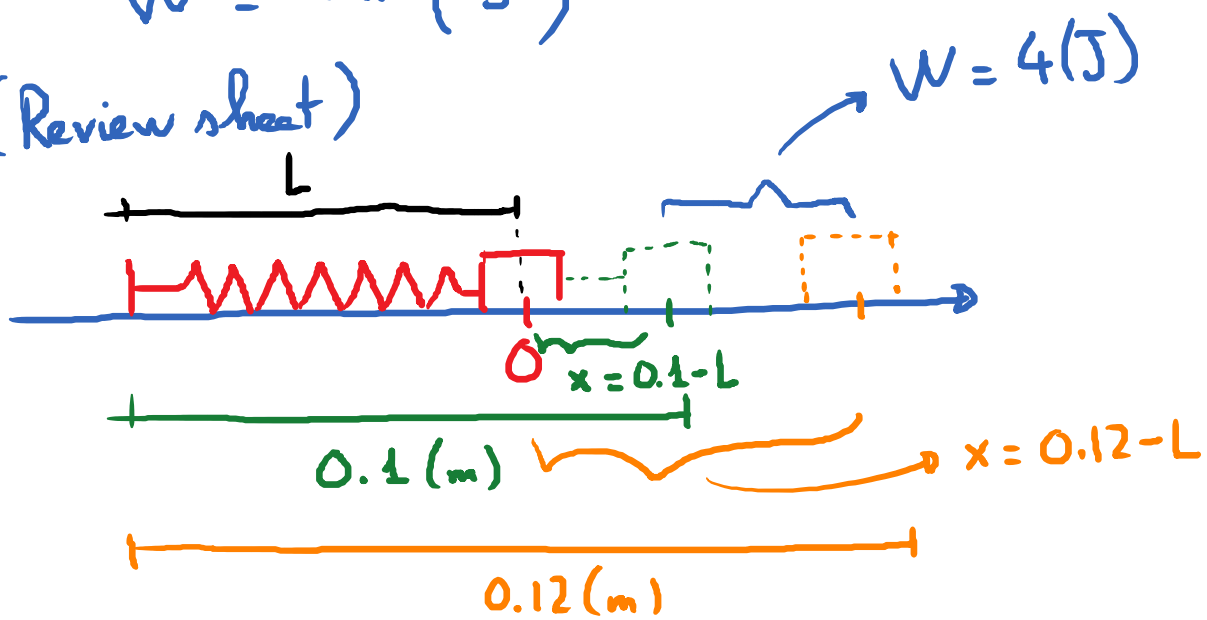
So, the formula for the force: $\boxed{F(x) = 800x}$

Work done in stretching spring from $x = 0.05$ to $x = 0.08$ is

$$W = \int_{0.05}^{0.08} F(x)\,dx = \int_{0.05}^{0.08} 800x\,dx = 800 \cdot \frac{x^2}{2}\Big|_{0.05}^{0.08}$$

$W = \ldots\ldots (J)$

#4 (Review sheet)



In stretching the spring from $x = 0.1 - L$ to $x = 0.12 - L$, we have:

$$4 = \int_{0.1-L}^{0.12-L} kx\,dx = k \cdot \frac{x^2}{2}\Big|_{0.1-L}^{0.12-L}$$

W

$$4 = \frac{k}{2}\left[(0.12-L)^2 - (0.1-L)^2\right]$$

$$8 = k \cdot \left[0.0144 - 0.24L + L^2 - 0.01 + 0.2L - L^2\right]$$

$$8 = k \cdot \left[0.0044 - 0.04L\right]$$

In stretching from $x = 0.12 - L$ to $x = 0.14 - L$, we have:

$$20 = \int_{0.12-L}^{0.14-L} kx\,dx = \frac{k}{2}x^2\Big|_{0.12-L}^{0.14-L}$$

$$40 = k \cdot \left[(0.14-L)^2 - (0.12-L)^2\right]$$

$$= k\left[0.0196 - 0.28L + L^2 - 0.0144 + 0.24L - L^2\right]$$

$$40 = k\left[0.0052 - 0.04L\right]$$

So, we have 2 equations:

$$8 = k\cdot\left[0.0044 - 0.04L\right] \rightarrow k = \frac{8}{0.0044 - 0.04L}$$

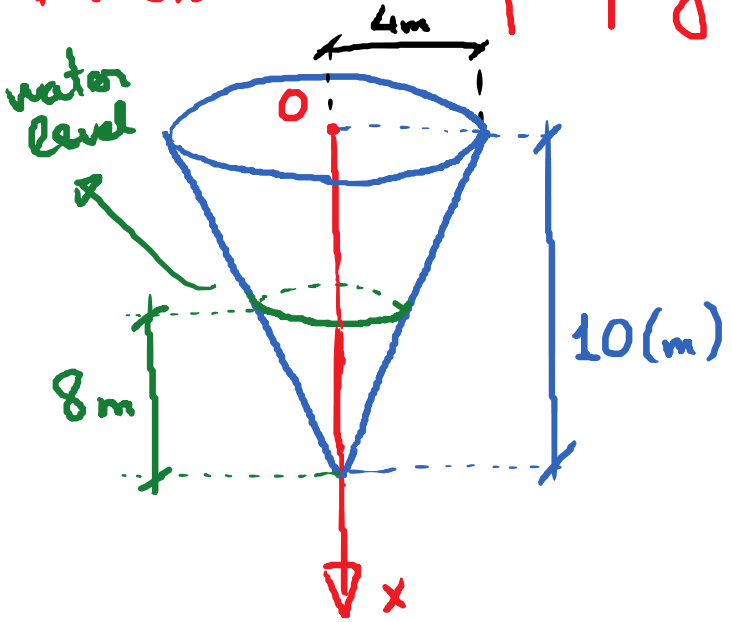
$$40 = k\cdot\left[0.0052 - 0.04L\right]$$

$$5 = \frac{0.0052 - 0.04L}{0.0044 - 0.04L}$$

$$5(0.0044 - 0.04L) = 0.0052 - 0.04L$$

$\longrightarrow$ .....

* Work done in pumping water (or liquid) out of a tank



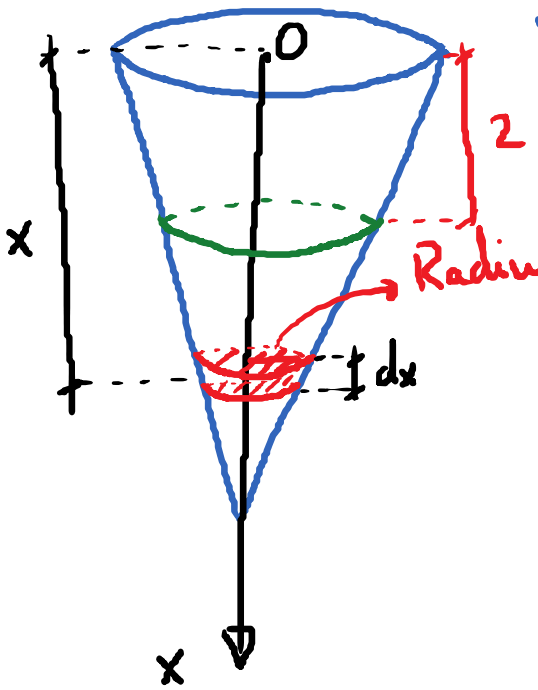
Tank: shape is an inverted cone as in the picture.

Density of water is 1000 kg/m^3

Find the work done to empty the tank by pumping water over the top of the tank

Strategy: * Find a formula for the work done in pumping a very "thin" slice of water out of the tank

* Integrate that formula along the entire body of water to find the total work done.



$$\text{Work}_{\text{small slice}} = (\underbrace{\text{force}_{\text{slice}}}_{\text{gravity}}) \cdot (\underbrace{\text{distance}}_{x})$$

$$= (m_{\text{slice}} \cdot g) \cdot x$$

$$= (\underbrace{\text{Volume}_{\text{slice}}}_{\text{mass}} \cdot \underbrace{\text{density}}_{1000} \cdot \underbrace{g}_{9.8}) \cdot x$$

$$= (\text{Base area}) \cdot (\underbrace{\text{thickness}}_{dx}) \cdot 9800x$$

$$= \underbrace{(\text{Base area}) \cdot (\text{thickness})}_{V_{\text{slice}}} \cdot 9800x$$

$$= \pi \cdot (\text{Radius})^2 \cdot 9800x\,dx$$

→ relate this to x