2.5. Physical Applications

Tuesday, September 11, 2018 8:09 AM

(1) Mass and Density

(2) Work done by a force

(3) Hydrostatic Force and Pressure

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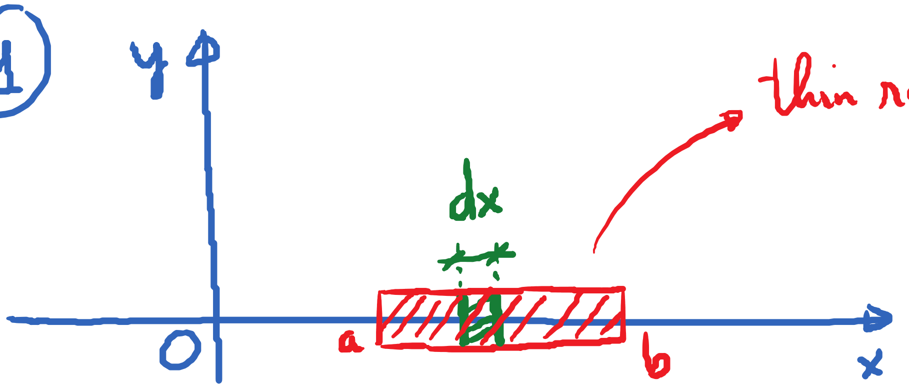
(1)



thin rod along x-axis
$a \le x \le b$.

Density at a point x is given by the function $\boxed{\rho(x)}$

non-constant density function

Q: Find m_{rod} (weight of the rod)

Density = mass per unit length (or area or volume)

Consider dx : a really small segment of the rod. Within this small segment, we can consider density to be constant and equal to $\rho(x)$.

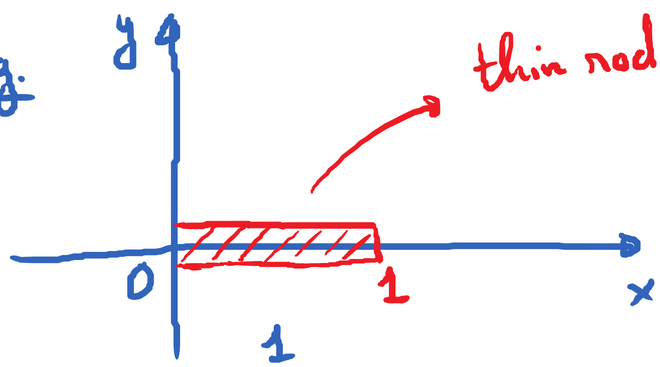
→ Mass of this small segment $= \underbrace{\rho(x)}_{\text{density}} \underbrace{dx}_{\text{length}}$

→ Mass of the entire rod $= \sum$ masses of these segments

So, $$\boxed{m_{rod} = \int_a^b \rho(x)\, dx}$$

Sum

E.g.



thin rod. Density function: $\rho(x) = \dfrac{5}{(x+2)^2}$

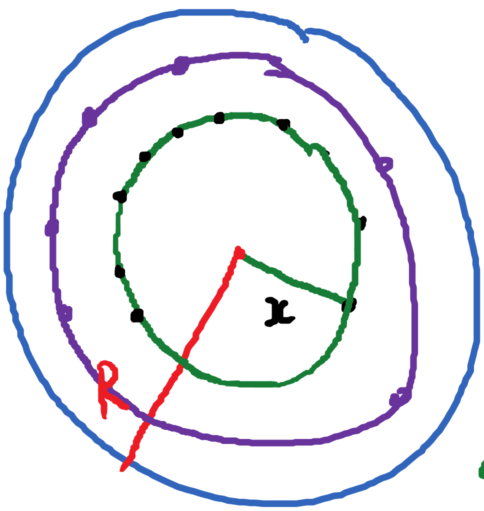
Q: Find m_{rod}?

$u = x+2$; $du = dx$

$$m_{rod} = \int_0^1 \frac{5}{(x+2)^2}\,dx = 5 \cdot \int_2^3 u^{-2}\,du = \left(-\frac{5}{u}\right)\Big|_2^3$$

$$= -\frac{5}{3} + \frac{5}{2} = \boxed{\frac{5}{6}}$$

* Circular Object with radial density.

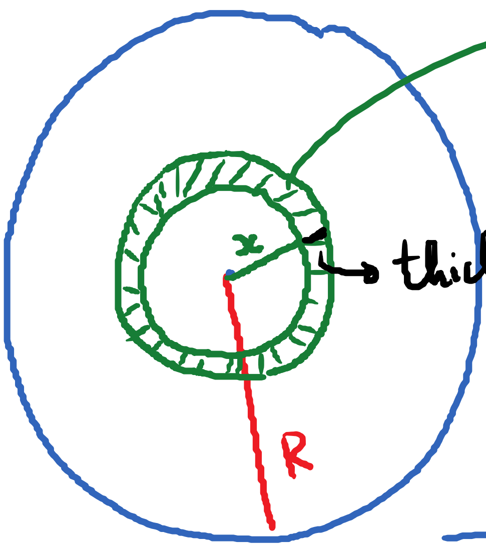


Disk of radius R

Radial density is given by a function $\rho(x)$

at every point on this circle, density $= \rho(x)$

→ <u>Q</u>: $m_{disk} = ?$



thickness $= dx$

mass of this circle $= \underbrace{2\pi \cdot x}_{\text{circumference}} \cdot \overbrace{\rho(x)}^{\text{density}}$

mass of the washer $= 2\pi \cdot x \cdot \rho(x) \cdot \overbrace{dx}^{\text{thickness}}$

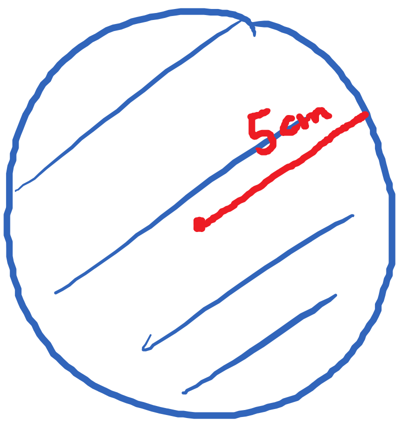
$$\rightarrow m_{disk} = \int_0^R 2\pi x \rho(x) dx$$

Formula for the mass of a disk with radius R with radial density $\rho(x)$:

$$\boxed{m = 2\pi \int_0^R x\,\rho(x)\,dx}$$

Ex. Disk

Radial density function $= e^{-x^2}$ (g/cm^2)



$\longrightarrow$ Find m_{disk}?

$$m = -2\pi \cdot \int_0^5 x\, e^{\boxed{-x^2}} \,(-dx)$$

$\rightarrow u$

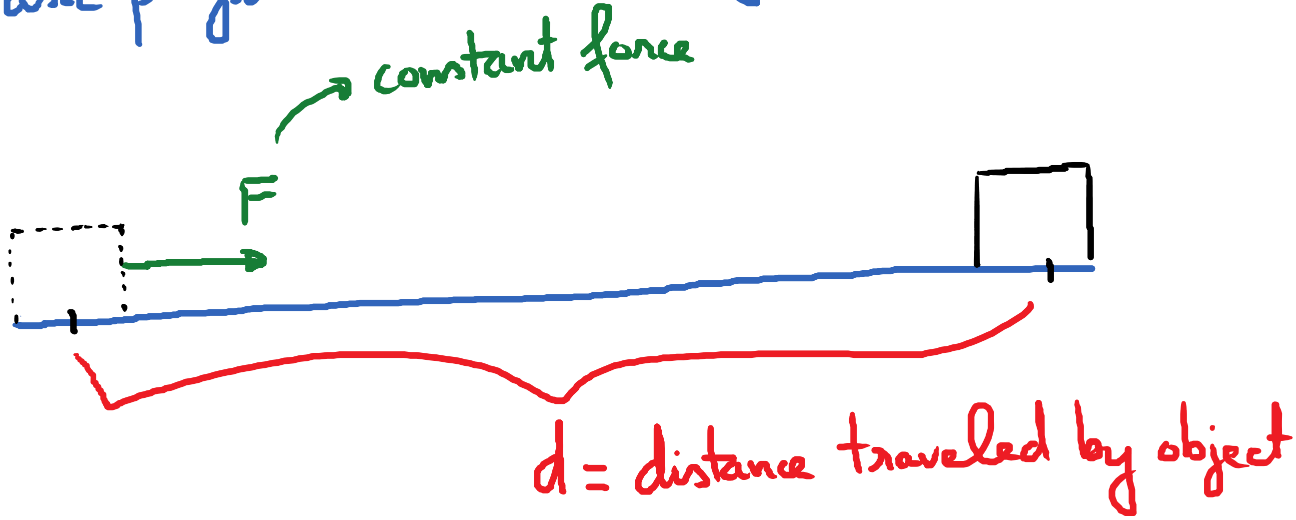
let $u = -x^2$. So, $du = -2x\,dx$

$$m = -\pi \int_0^{-25} e^u\,du = \pi \cdot \int_{-25}^{0} e^u\,du = \pi \cdot e^u \Big|_{-25}^{0}$$

$$= \boxed{\pi\left(1 - e^{-25}\right)}\ g$$

② Find work done by a variable force.

Basic physics: work done by a constant force.

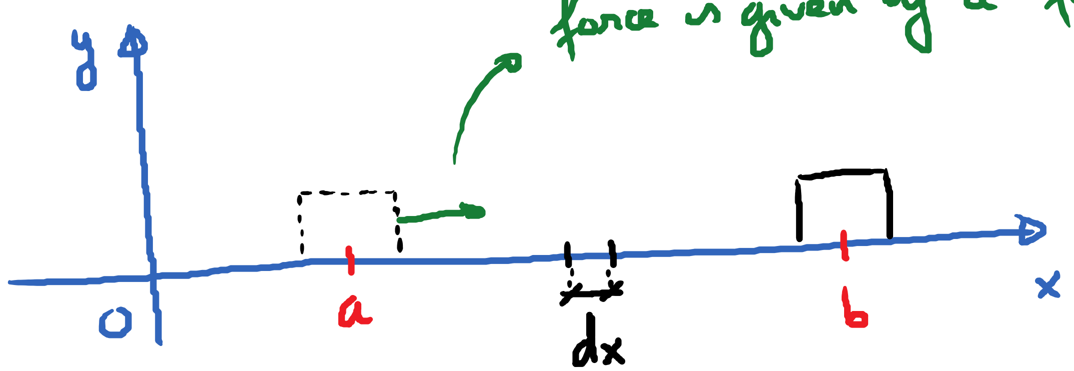


Work done by F = (Force)·(distance)

$$\boxed{W = F \cdot d}$$

What if the force is NOT constant?

force is given by a function $f(x)$



$W = ?$

Divide the distance from a to b into really small segments dx. We can consider the force to be constant within each such small segment.

Work done in moving the object a distance dx with force f(x) is: $\underbrace{f(x)}_{\text{force}} \cdot \underbrace{dx}_{\text{distance}}$

→ Total work done in moving object from a to b

= $\sum$ work done in moving the object in these small distances

$$= \int_a^b f(x)\,dx$$

$$\boxed{W = \int_a^b f(x)\,dx}$$