2.6. Moments and Center of Mass

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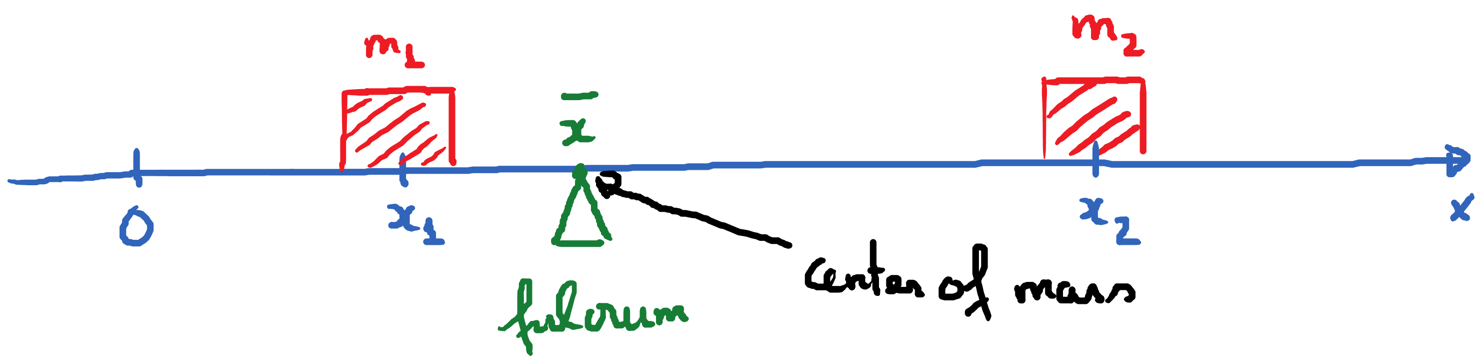
* 2 point masses m_1 and m_2 located on the x-axis at the x-coordinates x_1 and x_2

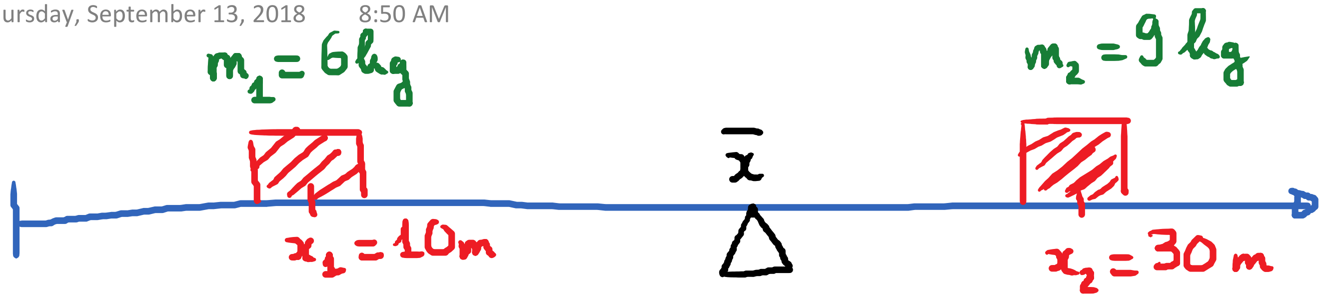


The center of mass of this system is the coordinate $\bar{x}$ of the point on the x-axis where we should place the fulcrum to make the system balanced.

The formula for $\bar{x}$ is the "weighted" average of x_1 and x_2

$$\boxed{\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

E.g.



Center of mass : $\bar{x} = \dfrac{6\cdot(10) + 9\cdot(30)}{6+9} = 22\,(m)$

In general, if we have n masses $m_1, m_2, \dots, m_n$ located at $x_1, x_2, \dots, x_n$ on the x-axis, then the center of mass $\bar{x}$ is given by :

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n}$$

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}}$$

moment of the system

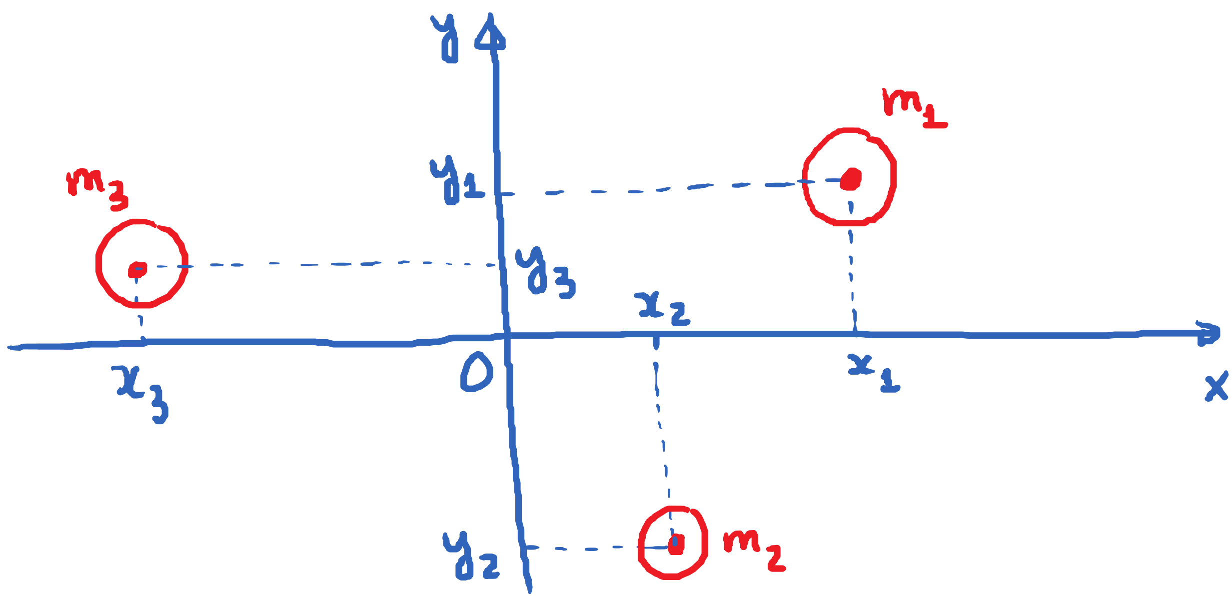
total mass of system

Note: The quantity:

$$M = \sum_{i=1}^{n} m_i x_i$$ is called the moment of the system

$$\boxed{\text{center of mass} = \frac{\text{Moment}}{\text{total mass}}}$$

Now, consider a system of n masses located at n points on the xy-plane : (x_1, y_1); (x_2, y_2); ... ; (x_n, y_n)



Picture for n = 3 masses

The center of mass of this system is the point $(\bar{x}, \bar{y})$

where:

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}}$$

$$m = \sum_{i=1}^{n} m_i = \text{total mass of the system.}$$

The quantity $\sum_{i=1}^{n} m_i x_i$ is called the y-moment of the system.

$$\boxed{M_y = \sum_{i=1}^{n} m_i x_i}$$

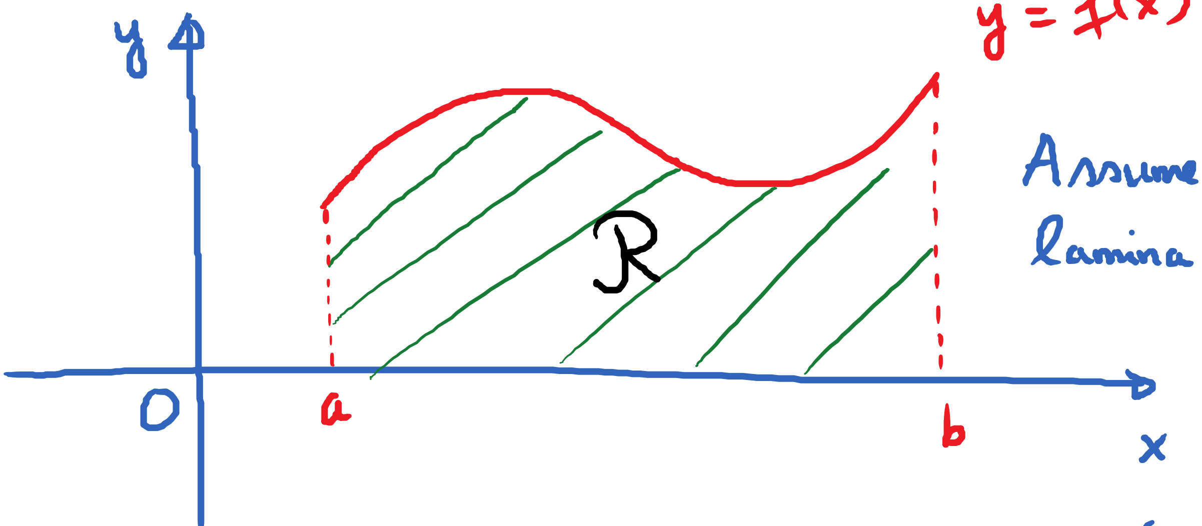
The quantity $\sum_{i=1}^{n} m_i y_i$ is called the x-moment of the system.

$$\boxed{M_x = \sum_{i=1}^{n} m_i y_i}$$

Formula for center of mass $(\bar{x}, \bar{y})$:

$$\boxed{\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}}$$

* Center of mass of thin plates (thin plate = lamina)



Assume that the lamina has uniform density ρ

Q: How do we find the center of mass $(\bar{x}, \bar{y})$ of this plate?

$$\bar{x} = \frac{M_y}{m} \; ; \quad \bar{y} = \frac{M_x}{m}$$

$$m = \underbrace{(\text{density})}_{\rho} \cdot (\text{Area})$$

$$\boxed{m = \rho \cdot \int_a^b f(x)\,dx}$$

y- moment (Moment of R about y-axis):

$$\boxed{M_y = \rho \cdot \int_a^b x f(x)\,dx}$$

x - moment (Moment of R about x-axis)

$$\boxed{M_x = \rho \cdot \int_a^b \frac{1}{2} \cdot [f(x)]^2\,dx}$$

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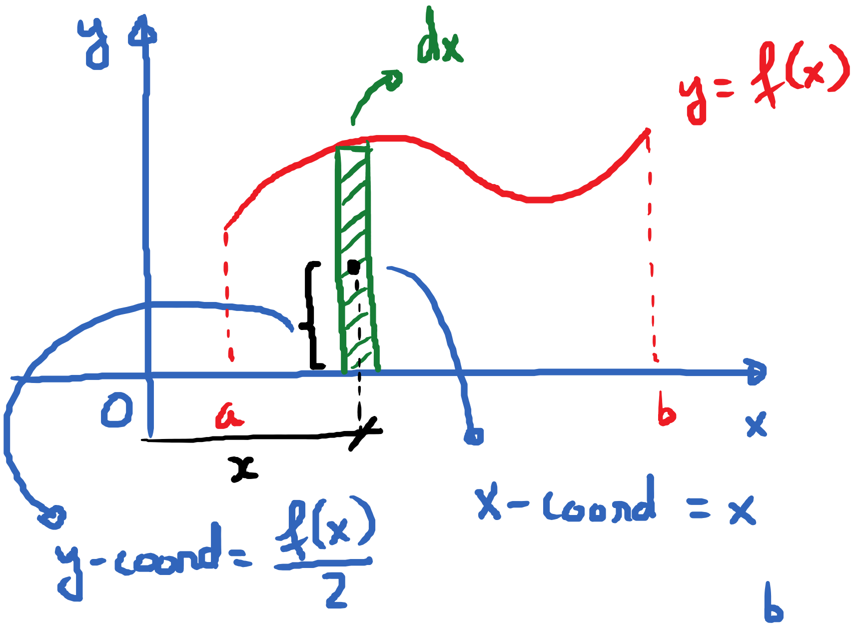
Then $\bar{x} = \dfrac{M_y}{m}$ and $\bar{y} = \dfrac{M_x}{m}$

Why is this formula true?



y-moment of strip

$= x \cdot \text{mass}$

$= x \cdot \text{area} \cdot \text{density}$

$= \rho \cdot x \cdot f(x) \cdot dx$

$\longrightarrow$ y-moment of system: $\displaystyle\int_a^b \rho x f(x)\,dx$

x-moment of strip = y-coord · mass

$$= \frac{f(x)}{2} \cdot f(x) \cdot dx \cdot \rho$$

$$= \frac{\rho}{2} \cdot [f(x)]^2\,dx$$