$$\int e^x \sin x \, dx$$

$$\begin{cases} u = \sin x & (\text{Trig. Function}) \\ dv = e^x \, dx & (\text{Exp. Function}) \end{cases}$$

T is before E in L.I.A.T.E

$$\begin{cases} du = \cos x \, dx \\ v = e^x \end{cases}$$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$\begin{cases} u = \cos x \\ dv = e^x dx \end{cases} \rightarrow \begin{cases} du = -\sin x dx \\ v = e^x \end{cases}$$

$$\int e^x \sin x \, dx = e^x \sin x - \left(e^x \cos x - \int e^x \cdot (-\sin x) \, dx \right)$$

$$\int e^x \sin x dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\boxed{\int e^x \sin x\,dx = \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + C}$$

2nd way: Tabular method : $\int e^x \sin x\,dx$

	D		I
u	$\sin x$	$+$	e^x
	$\cos x$	$-$	e^x
	$-\sin x$	$+$	e^x

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + \int e^x \cdot (-\sin x)\,dx$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$\rightarrow 2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x\,dx = \boxed{\frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + C}$$

Ex. Find the antiderivative

① $\int (\ln x)^2\, dx$ ② $\int x\cdot\tan^2(x)\, dx$

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① $\int (\ln x)^2\, dx$

Let $\begin{cases} u = (\ln x)^2 \\ dv = dx \end{cases}$ Then $\begin{cases} du = 2\cdot\frac{\ln x}{x}\, dx \\ v = x \end{cases}$

$$\int (\ln x)^2\, dx = x(\ln x)^2 - 2\int \cancel{x}\cdot\frac{\ln x}{\cancel{x}}\, dx$$

we did this

$$= x(\ln x)^2 - 2\int \ln x\, dx$$

$$= \boxed{x(\ln x)^2 - 2(x\ln x - x) + C}$$

* $\int (\ln x)^2 \, dx$

$$\begin{cases} u = \ln x \\ dv = \ln x \, dx \end{cases} \longrightarrow \begin{cases} du = \frac{1}{x} dx \\ v = x\ln x - x \end{cases}$$

$$\int (\ln x)^2 dx = \ln x \cdot (x \ln x - x) - \int (\ln x - 1) dx$$

$$= \ln x (x \ln x - x) - (x \ln x - x) + x + C$$

② $\int \underbrace{x}_{\text{algebraic}} \underbrace{\tan^2(x)}_{\text{trig}} dx$

Let $\begin{cases} u = x \\ dv = \tan^2(x) dx \end{cases}$

Then $\begin{cases} du = dx \\ v = \int \tan^2(x) dx \end{cases}$

$$v = \int (\sec^2(x) - 1) dx$$

$$v = \tan x - x$$

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$$\int x\tan^2(x)\,dx = x\cdot(\tan x - x) - \int(\tan x - x)\,dx$$

$$= x(\tan x - x) - \left[-\ln|\cos x| - \frac{x^2}{2}\right] + C$$

$$= x(\tan x - x) + \ln|\cos x| + \frac{x^2}{2} + C.$$

* Integration by parts for Definite Integrals

$$\boxed{\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du}$$

E.g. $\int_0^1 \arctan(x)\,dx$

$$\begin{cases} u = \arctan(x) \\ dv = dx \end{cases} \rightarrow \begin{cases} du = \dfrac{1}{1+x^2}dx \\ v = x \end{cases}$$

$$\int_0^1 \arctan(x)\,dx = x\arctan(x)\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{\overbrace{2x\,dx}^{du}}{\underbrace{1+x^2}_{u}}$$

$du = 2x\,dx$

$$= \arctan(1) - 0 - \frac{1}{2}\int_1^2 \frac{du}{u}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cdot \ln|u|\Big|_1^2$$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\ln(2) - \underbrace{\ln(1)}_{0}\right)$$

$$= \boxed{\frac{\pi}{4} - \frac{\ln(2)}{2}}$$

Ex.

Find $\displaystyle\int_4^9 \frac{\ln x}{\sqrt{x}}\,dx = \int_4^9 \overbrace{x^{-\frac{1}{2}}}^{\text{algebraic function}} \underbrace{\ln x}_{\text{Log function}}\,dx$

Let $\begin{cases} u = \ln x \\ dv = x^{-\frac{1}{2}}\,dx \end{cases}$ Then $\begin{cases} du = \frac{1}{x}\,dx \\ v = 2x^{\frac{1}{2}} \end{cases}$

$$\int_4^9 \frac{\ln x}{\sqrt{x}}\,dx = 2x^{\frac{1}{2}}\ln x\Big|_4^9 - 2\int_4^9 x^{\frac{1}{2}}\cdot\frac{1}{x}\,dx$$

$$= 6\ln(9) - 4\ln(4) - 2\int_4^9 x^{-\frac{1}{2}}\,dx$$

$$= 6\ln(9) - 4\ln(4) - 2\cdot 2x^{\frac{1}{2}}\Big|_4^9$$

$$= \boxed{12\ln(3) - 8\ln(2) - 4}$$