**Scenario 2:** Power of Sine is odd, i.e., $m = 2k+1$

Save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining part as an expression in cosine only.

Then use the u-sub $\boxed{u = \cos x}$.

**Scenario 3:** Power of Sine and of cosine are both even; i.e. both m and n are even

→ Use the Power Reduction Formula (may have to apply it a few times)

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2} \; ; \; \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$$

E.g. (a) $\int \tan^6 x \sec^4 x \, dx$

(b) $\int \tan^5 x \sec^7 x \, dx$

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(a) $\int \tan^6 x \cdot \sec^2 x \cdot \sec^2 x \, dx$

$$= \int \underbrace{\boxed{\tan^6 x}}_{u^6} \left(1 + \underbrace{\boxed{\tan^2 x}}_{u^2}\right) \underbrace{\boxed{\sec^2 x \, dx}}_{du}$$

Let $u = \tan x$. Then $du = \sec^2 x \, dx$

$$= \int u^6 (1 + u^2) \, du = \int (u^6 + u^8) \, du$$

$$= \frac{u^7}{7} + \frac{u^9}{9} + C = \boxed{\frac{\tan^7 x}{7} + \frac{\tan^9 x}{9} + C}$$

(b) $\int \tan^4 x \cdot \sec^6 x \cdot \sec x \tan x \, dx$

$$= \int \underbrace{(\sec^2 x - 1)^2}_{u^2-1} \cdot \overbrace{\boxed{\sec^6 x}}^{u^6} \overbrace{\boxed{\sec x \tan x \, dx}}^{du}$$

Let $u = \sec x$. Then $du = \sec x \tan x \, dx$

$$= \int (u^2 - 1)^2 \cdot u^6 \cdot du = \int (u^4 - 2u^2 + 1) u^6 \, du$$

$$= \int (u^{10} - 2u^8 + u^6) \, du = \frac{u^{11}}{11} - \frac{2u^9}{9} + \frac{u^7}{7} + C$$

$$= \boxed{\frac{\sec^{11} x}{11} - \frac{2\sec^9 x}{9} + \frac{\sec^7 x}{7} + C}$$

# Strategy for $\int \tan^m x \sec^n x \, dx$

**Scenario 1:** Power of secant is even.

Save 1 $\sec^2 x$ factor. Use $\sec^2 x = 1 + \tan^2 x$ to express the remaining part in terms of tangent. Then

let $\boxed{u = \tan x}$

**Scenario 2:** Power of tangent is odd

Save one $\tan x \sec x$ factor. Use $\tan^2 x = \sec^2 x - 1$ to express the remaining part in terms of secant.

Then let $\boxed{u = \sec x}$

**Other scenarios:** No clearcut strategy.

Thursday, September 27, 2018 9:53 AM

# Integrals of the form:

$$\int \sin(mx)\cos(nx)\,dx\ ; \quad \int \sin(mx)\sin(nx)\,dx\ ;$$

$$\int \cos(mx)\cos(nx)\,dx$$

To deal with these: Use Product-To-Sum Formula

$$\boxed{\begin{aligned} \sin(A)\cos(B) &= \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right] \\ \sin(A)\sin(B) &= \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right] \\ \cos(A)\cos(B) &= \frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right] \end{aligned}}$$

E.g. Find $\int \sin(4x)\cos(5x)\,dx$

Apply the first product-to-sum formula

$$\int \sin(4x)\cos(5x)\,dx = \frac{1}{2}\int \left(\sin(-x) + \sin(9x)\right) dx$$

$$\boxed{\begin{aligned}\sin(-\theta) &= -\sin\theta \\ \cos(-\theta) &= \cos\theta\end{aligned}}$$

$$= \frac{1}{2}\int \left(-\sin x + \sin(9x)\right) dx$$

$$= \boxed{\frac{1}{2}\left(\cos x - \frac{\cos(9x)}{9}\right) + C}$$

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**E.g.** $$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$$

$m, n$ are positive integers; $m \neq n$

$$\frac{1}{2}\int_{-\pi}^{\pi}\left(\sin\left[(m-n)x\right] + \sin\left[(m+n)x\right]\right)dx$$

$$\frac{1}{2}\left(\frac{-\cos\left[(m-n)x\right]}{m-n} - \frac{\cos\left[(m+n)x\right]}{m+n}\right)\Bigg|_{-\pi}^{\pi}$$

$$= 0$$

$0.1 \cdot x = 25$

$2$

$k \cdot \frac{0.01}{2} = 25$

$k = \frac{50}{0.01} = 5000$

$$\int_{0.01}^{0.11} 5000x\,dx$$

$$5000 \cdot \frac{x^2}{2} \Big|_{0.01}^{0.11}$$