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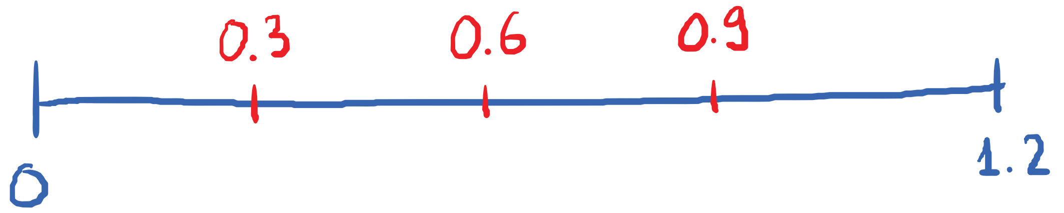
$$\boxed{T_4 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]}$$

Trapezoid Rule

4 subinterval

E.g. Estimate $\int_0^{1.2} \sin(x^2)\,dx$ using the Trapezoid Rule with $n = 4$ subinterval.



$$\Delta x = 0.3 \quad ; \quad \frac{\Delta x}{2} = 0.15$$

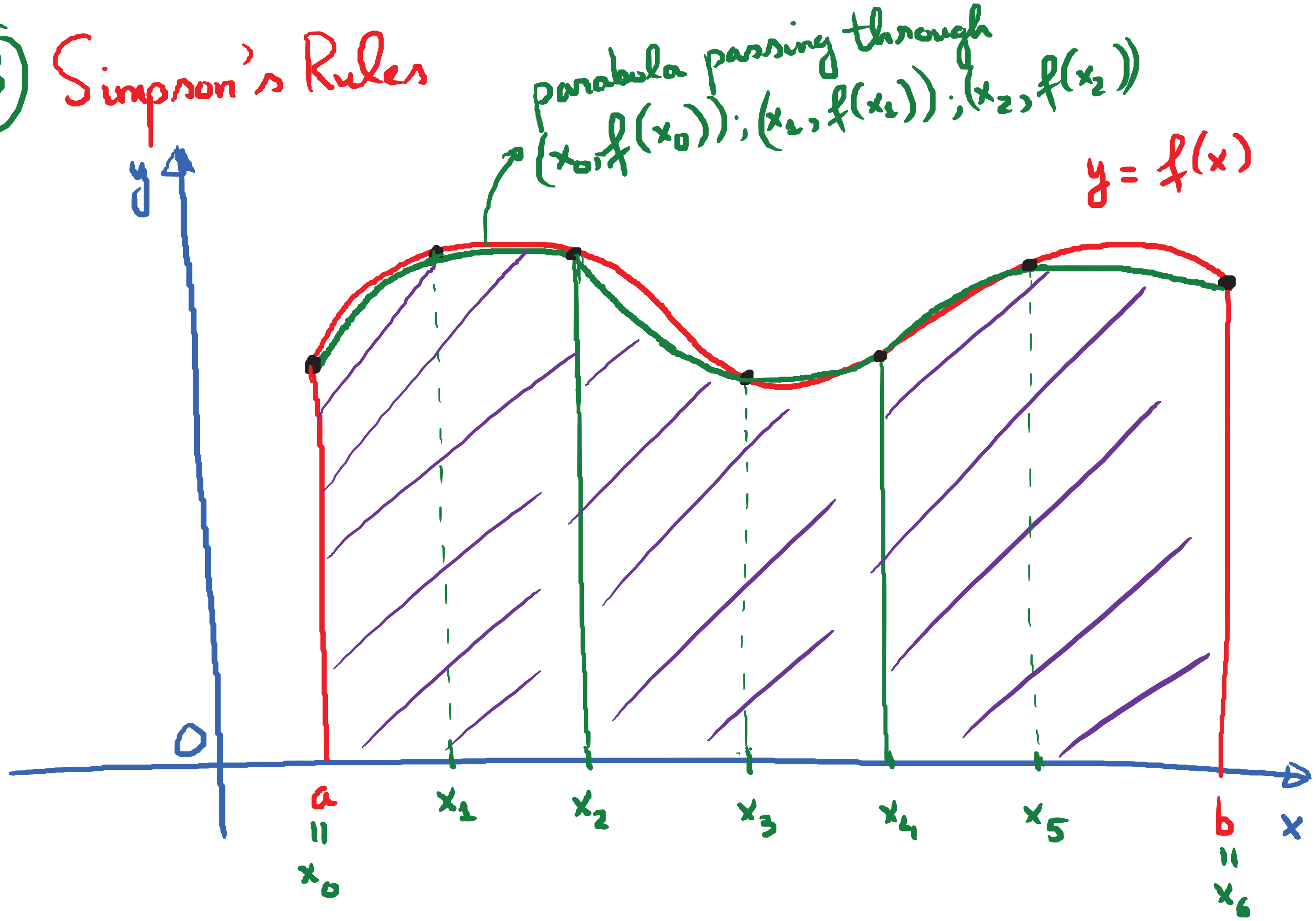
$$T_4 = 0.15\left[\sin(0^2) + 2\sin(0.3^2) + 2\sin(0.6^2) + 2\sin(0.9^2) + \sin(1.2^2)\right]$$

$$T_4 \approx 0.49865$$

In general, the formula for the Trapezoid rule with n trapezoids is

$$\boxed{\begin{gathered} T_n = \frac{\Delta x}{2}\Big[f(\overset{a}{x_0}) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(\overset{b}{x_n})\Big] \\ \Delta x = \frac{b-a}{n} \end{gathered}}$$

③ Simpson's Rules



Simpson's Rule for $n = 6$ subintervals

$$S_6 = \text{sum of areas under 3 parabolas}$$

$$= \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right]$$

n

Simpson's Rule

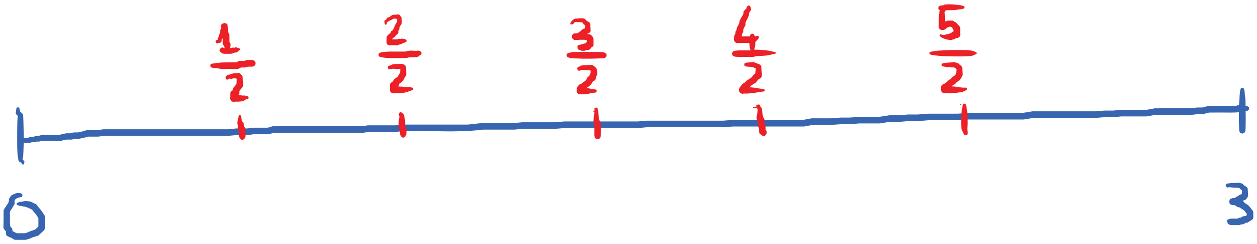
In general, the formula for Simpson's rule with n subintervals (n must be even) is:

$$\boxed{S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]}$$

E.g. Use Simpson's Rule with $n=6$ to estimate the integral:

$$\int_0^3 \frac{dx}{1+x^3}$$

$$\Delta x = \frac{3}{6} = \frac{1}{2}$$



$$T_6 = \frac{1}{6}\left[f(0)+4f\left(\tfrac{1}{2}\right)+2f(1)+4f\left(\tfrac{3}{2}\right)+2f(2) + 4f\left(\tfrac{5}{2}\right)+f(3)\right]$$

$$\simeq 1.1614$$

④ Error Estimates for these numerical integration methods

Estimate $\int_a^b f(x)\,dx$ using

the Midpoint Rule with n subintervals : M_n

— Trapezoid Rule with n subintervals : T_n

— Simpson's Rule ——— : S_n

Exact errors for each of these methods :

* Exact error for Midpoint Rule:

$$E_{M_n} = \underbrace{\int_a^b f(x)\,dx}_{\text{actual value of the integral}} - \underbrace{M_n}_{\text{estimate using Midpoint}}$$

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Exact error for Trapezoid: $\boxed{E_{T_n} = \int_a^b f(x)dx - T_n}$

———— Simpson: $\boxed{E_{S_n} = \int_a^b f(x)dx - S_n}$

We cannot find these exact errors in practice.

→ Find upper bounds for these errors.

Formulas for the upper bounds of E_{M_n}, E_{T_n}, E_{S_n}

$$\boxed{\left|E_{M_n}\right| \leq \frac{K\cdot(b-a)^3}{\underbrace{24n^2}_{\text{upper bound for the error}}}}$$