5.1. Sequences

Tuesday, October 16, 2018 8:06 AM

E.g. $1, 1, 2, 3, 5, 8, 13, \ldots$ Fibonacci Sequence

A sequence is a list of numbers with a particular order.

In general, the numbers in a sequence are denoted by

$$\underbrace{a_1}_{\text{first term}}, \underbrace{a_2}_{\text{second term}}, a_3, \ldots, \underbrace{a_n}_{n^{\text{th}}\text{ term or the general term}}, \ldots$$

The term right before a_n is a_{n-1}.

____________ after a_n is a_{n+1}.

Notation for the sequence: $\{a_1, a_2, \ldots\}$ or $\{a_n\}_{n=1}^{\infty}$

E.g. Many sequences are given by a formula that describes a_n; i.e., $a_n = f(n)$

(a) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$; $\left(a_n = \frac{n}{n+1} \text{ ; } n = 1, 2, \ldots\right)$

$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$

(b) $\left\{\frac{(-1)^n (n+1)}{3^n}\right\}_{n=1}^{\infty}$; $a_n = \frac{(-1)^n (n+1)}{3^n}$; $n \geq 1$

$\left\{-\frac{2}{3}, \frac{1}{3}, -\frac{4}{27}, \frac{5}{81}, \ldots\right\}$

(c) $\left\{\sqrt{n-3}\right\}_{n=3}^{\infty}$; $a_n = \sqrt{n-3}$; $n \geq 3$

$\left\{0, 1, \sqrt{2}, \sqrt{3}, \ldots\right\}$

E.g. Find a formula for the general term a_n of the sequence $\left\{\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \ldots\right\}$

Tuesday, October 16, 2018 8:22 AM

$$\left\{ (-1)^{n+1} \cdot \frac{n+2}{5^n} \right\}_{n=1}^{\infty} \quad ; \quad a_n = (-1)^{n+1} \cdot \frac{n+2}{5^n}.$$

E.g. Many sequences are given recursively.

Fibonacci sequence: $1, 1, 2, 3, 5, 8, \ldots$

$$\left.\begin{array}{l} a_n = a_{n-1} + a_{n-2} \ ; \ n \geqslant 3 \\ a_1 = 1 \ ; \ a_2 = 1 \end{array}\right\} \text{Recursively defined}$$

Q: Is there a formula for a_n as a function of n?

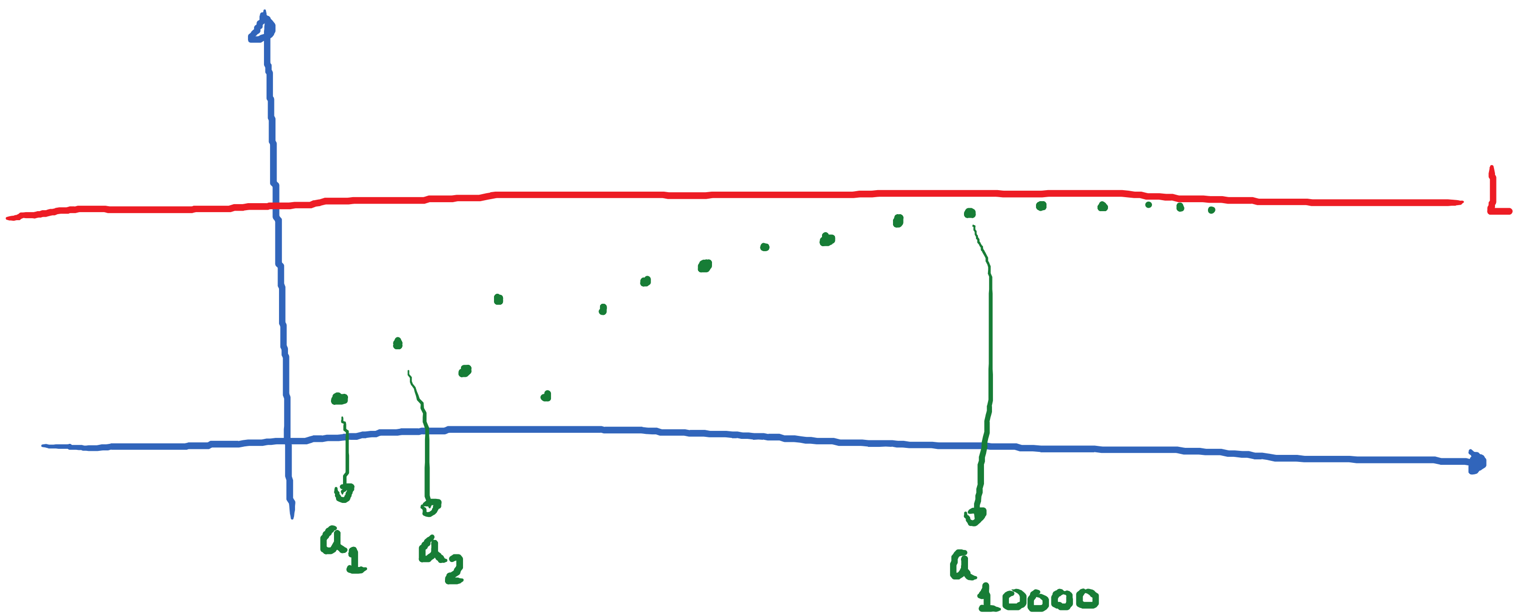
$(a_n = f(n))$

Yes. $a_n = \frac{1}{\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n$

Tuesday, October 16, 2018 8:30 AM

Limit of a Sequence

The notation: $\lim_{n\to\infty} a_n = L$ means that the terms of the sequence $\{a_n\}$ approach L as n becomes large.



<u>Note</u>: If a_n is given by a function of n, i.e., $a_n = f(n)$, then $\lim_{n\to\infty} a_n = \lim_{n\to\infty} f(n)$

E.g. $a_n = \frac{1}{n}$; $n \geq 1$. Find $\lim_{n\to\infty} a_n$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0.$$

E.g. $a_n = \frac{3n}{7n+1}$; $n \geq 1$. Find $\lim_{n\to\infty} a_n$

$\frac{\infty}{\infty}$ L'Hopital

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n}{7n+1} = \lim_{n\to\infty} \frac{3}{7} = \boxed{\frac{3}{7}}$$

E.g. $a_n = \frac{n}{\sqrt{17+n}}$; $n \geq 1$. Find $\lim_{n\to\infty} a_n$.

as n is large a_n behaves like : $\frac{n}{\sqrt{n}} = \sqrt{n}$

So, $\lim_{n\to\infty} a_n = \infty$

E.g. $a_n = \frac{\ln(n)}{n}$; $n \geq 1$. Find $\lim\limits_{n\to\infty} a_n$.

$$\lim_{n\to\infty} \frac{\ln(n)}{n} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{1}{n} = \boxed{0}$$

E.g. $a_n = (-1)^n$; $n \geq 1$. Find $\lim\limits_{n\to\infty} a_n$.

$$\{-1, 1, -1, 1, -1, 1 \cdots\}$$

So, $\lim\limits_{n\to\infty} a_n$ DNE.

E.g. $a_n = \frac{(-1)^n}{n}$; $n \geq 1$. Find $\lim\limits_{n\to\infty} a_n$.

Squeeze Theorem: $\frac{-1}{n} \leq a_n \leq \frac{1}{n}$

$$0$$

$$\lim_{n\to\infty} a_n = 0$$