Absolute Convergence means: both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|$ converge.

<u>Note</u>: $\sum_{n=1}^{\infty} |a_n|$ converges will imply that $\sum_{n=1}^{\infty} a_n$ converges. So, if we can demonstrate that the "absolute value series" converges, we can conclude that the original series converges absolutely.

Error Estimates for Alternating Series

Given an alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$.

Suppose that it converges by the A.S.T.

Let say it converges to a number s.

So, $s = \sum_{n=1}^{\infty} (-1)^{n+1} b_n$.

We want to use the partial sums of the series to estimate s. Say, we use the sum of the first N terms to estimate s.

$$\underbrace{S_N}_{N^{\text{th}} \text{ partial sum.}} = \sum_{n=1}^{N} (-1)^{n+1} b_n$$

Q: How good is this approximation S_N to s?

$\longleftrightarrow$ How far away is S_N to s; i.e., can we find bounds for $|s - S_N|$?

Thm about error estimate for A.S.

$$\boxed{|s - S_N| \le b_{N+1}}$$

E.g. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \boxed{\frac{1}{n}} \longrightarrow b_n = \frac{1}{n}$.

Suppose that I use the first 10 terms; i.e., I use S_{10} (sum of) to estimate the infinite sum.

Find an upper bound for the error.

Ans: $\boxed{\dfrac{1}{11}}$

$$S_{10} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots + \frac{1}{10} \approx \boxed{-0.6456}$$

approximation

It is know that $s = \boxed{-\ln(2)}$ → infinite sum

Error: $\underbrace{\left| -\ln(2) - (-0.6456) \right|}_{\text{error} = 0.0475} < \underbrace{\frac{1}{11}}_{0.09}$

<u>Ex.</u> How many terms should I use in a partial sum to approximate the infinite sum:

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^2} \qquad b_n$$

so that the error is no more than 0.001?

Suppose I use S_N to approximate the inf. sum.

$$|s - S_N| \leq b_{N+1} = \frac{1}{(N+1)^2}$$

For error ≤ 0.001, require $\frac{1}{(N+1)^2} \leq 0.001$

$$(N+1)^2 \geq 1000 \rightarrow N \geq \sqrt{1000} - 1 \approx 30.62$$

So, we can choose N to be 31.