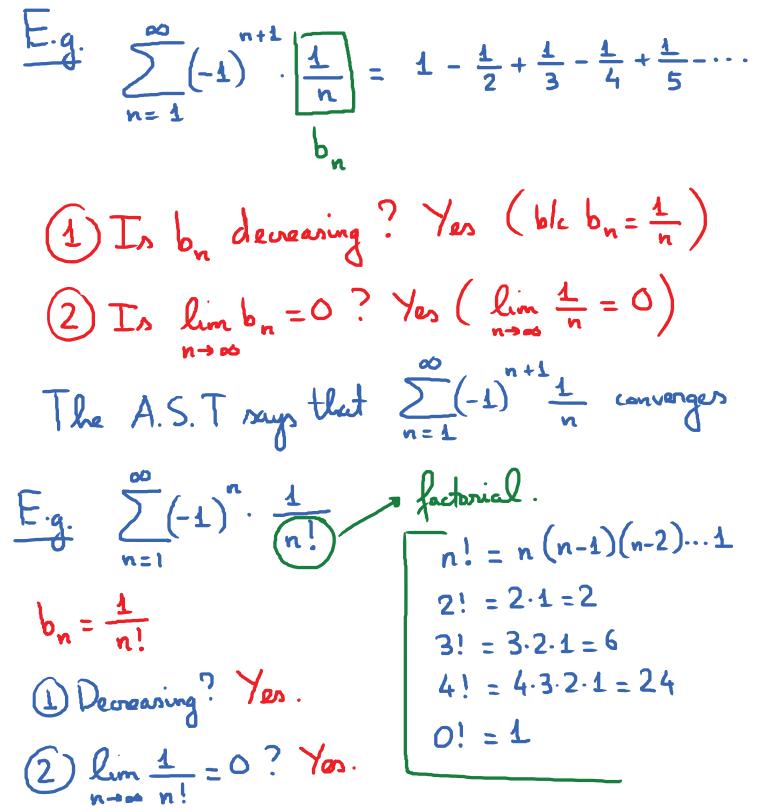
5.5. Alternating Series Test Tuesday, October 23, 2018 10:00 AM

A.S.T. (riven an alternating revies of the form $\sum_{n=1}^{n} (-1)^n b_n$ on $\sum_{n+1}^{\infty} (-1)^{n+1} b_n$ with $b_n \ge 0$. If the sequence (b_n) satisfies the conditions: (1) $b_n \ge b_{n+1}$ for all $n \ge N$ (it is a decreasing sequence) $(2) \lim_{n \to \infty} b_n = 0$ Then the alternating series converges.

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E

By the A.S.T.
$$\sum_{n=1}^{\infty} (-1)^{n} \cdot \frac{1}{n!} \text{ converges.}$$

E.g.
$$\sum_{n=1}^{\infty} (-1)^{n+4} \cdot \frac{n^{2}}{2n^{3}+1} \quad \Rightarrow \text{Altennating Series}$$

b_n = $\frac{n^{2}}{2n^{3}+1}$
1) Is b_n decreasing?
b_n = $\frac{n^{2}}{2n^{3}+1} \quad \Rightarrow \text{function associated with bn in}$
quotient suba $f(n) = \frac{n^{2}}{2n^{3}+1}$
f³(n) $\frac{1}{2} \cdot \frac{(2n^{3}+1)(2n-n^{2}(6n^{2}))}{(2n^{3}+1)^{2}} = \frac{4n^{4}+2n-6n^{4}}{(2n^{3}+1)^{2}}$
 $= \frac{2n-2n^{4}}{(2n^{3}+1)^{2}} = \frac{2n(4-n^{3})}{(2n^{3}+1)^{2}} \leq 0 \text{ when } n \geq 1$

Thursday, November 1, 2018 8:10 AM

So,
$$f(n)$$
 is decreasing on $[1,\infty)$.
Hence, b_n is a decreasing sequence.
2) Is $\lim_{n\to\infty} b_n = 0$?
 $\lim_{n\to\infty} \frac{n^2}{2n^3 + 1} = 0$ (deg top < deg bottom)
By A.S.T., the series converges.
The concepts of Absolute convergence and (orditional
(onvergence.
E.g. Consider the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \cdots$
By the A.S.T. this series converges.

