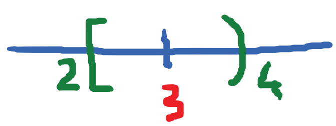
Test $x=4$: $\displaystyle\sum_{n=1}^{\infty} \frac{(4-3)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$

p-series with $p=1$, so diverges.

Test $x=2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(2-3)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by the Alternating Series Test.

So, I.O.C $= [2,4)$; R.O.C $= 1$



(b) Root Test: $\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{|x-6|^n}{n^n}}$ (Regardless of x)

$\displaystyle = \lim_{n\to\infty} \frac{|x-6|}{n} = 0 < 1.$

So, the series converges for all values of x.

I.O.C. $= (-\infty, \infty)$; R.O.C $= \infty$

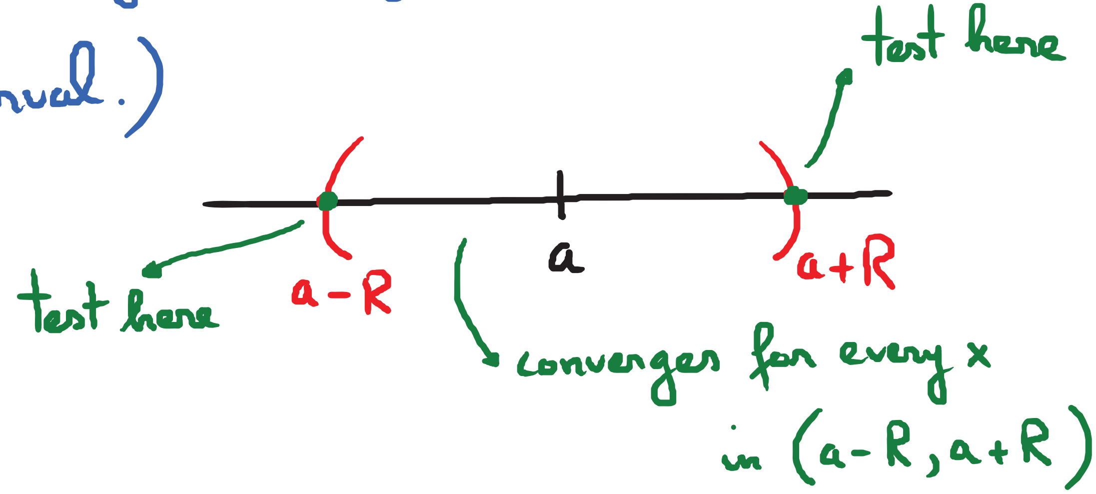
Theorem: Given any power series $\sum_{n=0}^{\infty} c_n (x-a)^n$

Exactly one of the following scenario will happen.

① The series converges only when $x = a$.
For every value $x \neq a$, it diverges. $(R.O.C = 0)$

② The series converges for all values of x. $(R.O.C = \infty)$

③ The series converges for every value of x within an interval surrounding the center a.
(It may converge or may diverge at the endpoints of that interval.)



E.g. Given: $\sum_{n=0}^{\infty} c_n 4^n$ converges.

Does the following series converge or diverge or undecided?

ⓐ $\sum_{n=0}^{\infty} c_n (-2)^n$ ⓑ $\sum_{n=0}^{\infty} c_n (-4)^n$.

Yes Undecided

E.g. Given: $\sum_{n=0}^{\infty} c_n x^n$ — converges when $x=-4$; diverges when $x=6$

What can be said about the following series

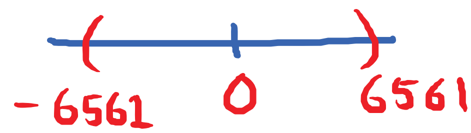
ⓐ $\sum_{n=0}^{\infty} c_n$ C ⓑ $\sum_{n=0}^{\infty} c_n 8^n$ D ⓒ $\sum_{n=0}^{\infty} c_n (-3)^n$ C

ⓓ $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$ D

E.g. $\sum_{n=0}^{\infty} c_n x^n$. Radius of convergence is 6561

What is the radius of convergence of $\sum_{n=0}^{\infty} c_n x^{2n}$?

$\sum_{n=0}^{\infty} c_n x^n$; For every x with $|x| < 6561$, it converges



$$\sum_{n=0}^{\infty} c_n x^{2n} = \sum_{n=0}^{\infty} c_n \left(\boxed{x^2}\right)^n; \quad |x^2| < 6561$$

$$|x| < \sqrt{6561} = \boxed{81}$$

Represent Functions by Power Series.

$$\underbrace{\frac{1}{1-x}}_{\text{Function}} = 1 + x + x^2 + x^3 + \cdots = \underbrace{\sum_{n=0}^{\infty} x^n}_{\text{Series}} \; ; \; \underbrace{-1 < x < 1}_{|x|<1}$$

E.g. Represent the function $\frac{1}{1+x^2}$ by a power series?

Find the I.O.C.

$$\frac{1}{1+x^2} = \frac{1}{1-\left(\boxed{-x^2}\right)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$$

$$|-x^2| < 1 \leftrightarrow |x|^2 < 1 \leftrightarrow |x| < 1.$$

(center $=0$)

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E.g. Represent $\frac{1}{2+x}$ by a power series. Find I.O.C.

$$\frac{1}{2+x} = \frac{1}{2\left(1+\frac{x}{2}\right)} = \frac{1}{2}\cdot\frac{1}{1-\boxed{\left(-\frac{x}{2}\right)}}$$

$$= \frac{1}{2}\cdot\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n = \frac{1}{2}\cdot\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{2^n}x^n$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}x^n$$

$\left|-\frac{x}{2}\right| < 1$

$|x| < 2$

I.O.C

E.g. Find a power series representation of the given function and find I.O.C.

(a) $f(x) = \frac{x}{9+x^2}$

(b) $f(x) = \frac{3}{x^2-x-2}$.

(a) $$\frac{x}{9+x^2} = x \cdot \frac{1}{9\left(1+\frac{x^2}{9}\right)} = \frac{x}{9} \cdot \boxed{\frac{1}{1-\boxed{\left(-\frac{x^2}{9}\right)}}}$$

$$= \frac{x}{9} \cdot \sum_{n=0}^{\infty} \left(-\frac{x^2}{9}\right)^n = \frac{x}{9} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{9^n} \cdot x^{2n}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{9^{n+1}} x^{2n+1}}$$

$$\left|-\frac{x^2}{9}\right| < 1 \leftrightarrow |x|^2 < 9 \leftrightarrow \boxed{|x| < 3}$$

(b) $$\frac{3}{x^2-x-2} = \frac{3}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$A = 1$; $B = -1$. $$\frac{1}{-2+x} - \frac{1}{1+x}$$

$$= \frac{1}{-2\left(1-\frac{x}{2}\right)} - \frac{1}{1-(-x)}$$

$\left|\frac{x}{2}\right| < 1 \rightarrow |x| < 2$

$|x| < 1$

$\boxed{|x| < 1}$

$$= -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n - \sum_{n=0}^{\infty}(-x)^n$$

$$= \sum_{n=0}^{\infty}\frac{-1}{2^{n+1}}\cdot x^n - \sum_{n=0}^{\infty}(-1)^n x^n$$

$$= \boxed{\sum_{n=0}^{\infty}\left[-\frac{1}{2^{n+1}} - (-1)^n\right]x^n}$$