6.4. Working with Taylor and Maclaurin series

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Recall:

$f(x)$	Series	I.O.C
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$	$(-\infty, \infty)$
$\sin x$	$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$	$(-\infty, \infty)$
$\cos x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots$	$(-\infty, \infty)$

$\boxed{\#8}$

$$\sin(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{\frac{2n+1}{2}}}{(2n+1)!}$$

$$\frac{\sin(\sqrt{x})}{\sqrt{x}} = \frac{1}{\sqrt{x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n x^{\frac{2n+1}{2}}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{\frac{2n+1}{2}}}{x^{\frac{1}{2}}(2n+1)!}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}} \qquad \text{I.O.C. } (0, \infty)$$

$\boxed{\#10}$ $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$$\cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

$$\frac{1}{2}\cos(2x) = \sum_{n=0}^{\infty} \frac{1}{2} \cdot \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

$$\frac{1}{2} - \frac{1}{2}\cos(2x) = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{1}{2} \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

$$= \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - \sum_{n=1}^{\infty} \frac{1}{2} \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 2^{2n-1} \cdot x^{2n}}{(2n)!}$$

$\boxed{\#2}$ a

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{4x^2} = \sum_{n=0}^{\infty} \frac{(4x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n!}$$

$$\boxed{e^{4x^2} = 1 + 4x^2 + 8x^4 + \frac{64x^6}{6} + \cdots}$$

$$T_3(x) = 1 + 4x^2$$

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a $T_3(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$

$$c_0 = \boxed{f(0)}^{=1}; \quad c_1 = \boxed{f'(0)}^{=0}; \quad c_2 = \boxed{\frac{f''(0)}{2}}^{=4}; \quad \boxed{c_3 = \frac{f'''(0)}{6}} = 0$$

$$f(x) = e^{4x^2}; \quad f'(x) = 8xe^{4x^2}$$

$$f''(x) = 8e^{4x^2} + 64x^2e^{4x^2}$$

$$f'''(x) = 64xe^{4x^2} + 128xe^{4x^2} + 512x^3e^{4x^2}$$

$$\boxed{T_3(x) = 1 + 4x^2}$$

$$\underline{e^{4x^2}} \cdot \underline{(192x + 512x^3)}$$

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(b) $|R_3(x)| \leq \frac{M}{4!} \cdot |x - a|^4 = \frac{M}{4!}(0.1)^4$

0.1 0

M: upper bound for $|f^4(x)|$ on $[0, 0.1]$

(3) $|R_n(x)| \leq \boxed{\frac{M}{(n+1)!} \cdot |0.1|^{n+1}}$

M is upper bound for $|\underbrace{f^{(n+1)}(x)}|$ on $[0, 0.1]$

$e^x \rightarrow$ largest $= e^{0.1}$ M

on $[0, 0.1]$

$$\frac{e^{0.1}}{(n+1)!}(0.1)^{n+1} < 0.0001$$

$\boxed{n = 3}$ 0.0000046

4 terms

4

$$\left|\frac{x^7}{7}\right| < 0.0005$$

$$|x| < 0.446$$

$$-0.446 < x < 0.446$$