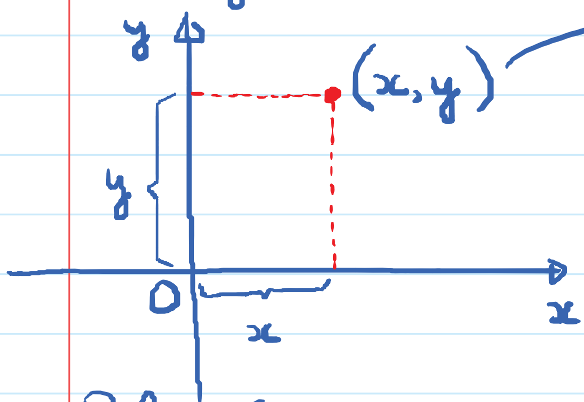
Tuesday, December 4, 2018 8:09 AM

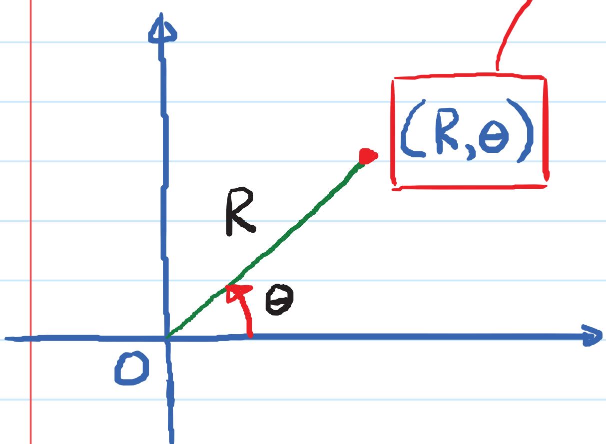
7.3 and 7.4. Polar Coordinates and the Calculus of Polar Curves

Rectangular Coordinates:



→ rectangular coordinates of the point

Polar Coordinates:



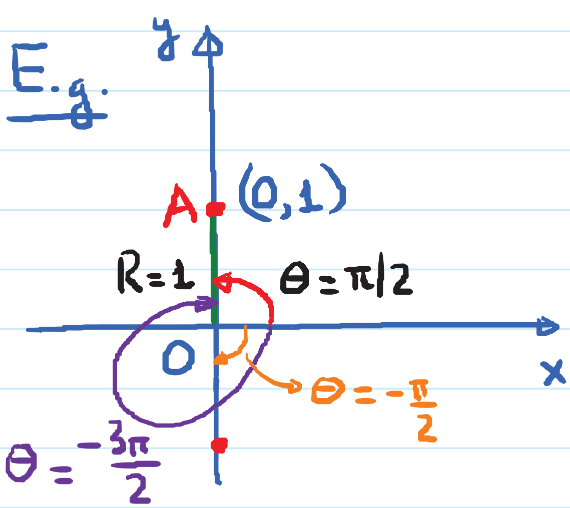
→ Polar Coordinates of point

Step 1: Draw a line segment from O to point.

Step 2: Measure the length of that line segment

Step 3: Measure the angle from positive part of x-axis to the ray connecting O to point

E.g.



Rectangular coord. of A : $(0,1)$

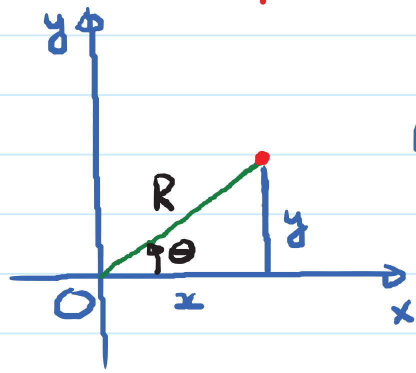
Polar coord. of A : $\left(1, \frac{\pi}{2}\right)$

or $\left(1, -\frac{3\pi}{2}\right)$

or $\left(-1, -\frac{\pi}{2}\right)$

* The polar coordinates representation of a point is not unique.

Relationship between Rectangular and Polar Coordinates



Polar → Rectangular

Given (R, θ) → $x = ?$, $y = ?$

$$x = R\cos\theta \; ; \; y = R\sin\theta$$

Rectangular → Polar

Given (x, y) → $R = ?$; $\theta = ?$

$$R = \sqrt{x^2 + y^2} \; ; \; \theta = \arctan\left(\frac{y}{x}\right)$$

Note: $\arctan(\cdot)$ only gives angles in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

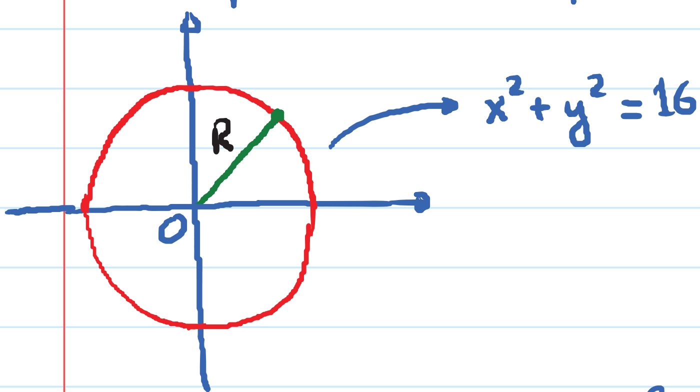
So, we need to add π to result to get the correct quadrant.

(x, y) in Q II $(x < 0; y > 0)$: $\theta = \pi + \arctan\left(\frac{y}{x}\right)$

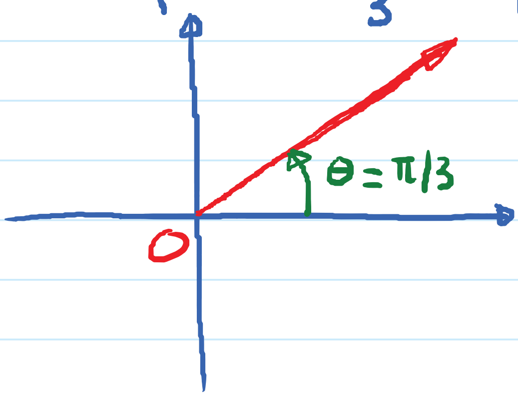
(x, y) in Q IV $(x < 0; y < 0)$: $\theta = \pi + \arctan\left(\frac{y}{x}\right)$

Graphs of equations in polar coordinates:

* Graph $R = 4$ in polar coordinates



* Graph $\theta = \frac{\pi}{3}$ in polar coordinates



Calculus with polar curves

(1) Tangent lines with polar curves

Given a polar curve whose equation is $R = f(\theta)$

Slope of the tangent line to this curve at a point.

$$\text{Slope} = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

Know: $y = R\sin\theta = f(\theta)\cdot\sin\theta \longrightarrow f'(\theta)\sin\theta + f(\theta)\cos\theta$

$x = R\cos\theta = f(\theta).\cos\theta \longrightarrow f'(\theta)\cos\theta - f(\theta)\sin\theta$

Another way to write this formula:

$$\frac{dy}{dx} = \frac{\frac{dR}{d\theta} \cdot \sin\theta + R\cos\theta}{\frac{dR}{d\theta}\cos\theta - R\sin\theta}$$

E.g. Consider the polar curve given by the equation

$$R = 1 + \sin\theta$$

Find the equation of the tangent line to the curve at the point where $\theta = \frac{\pi}{3}$.

Sol: $\theta = \frac{\pi}{3}$; $R = 1 + \sin\left(\frac{\pi}{3}\right) = 1 + \frac{\sqrt{3}}{2}$

$$\frac{dy}{d\theta} = \frac{dR}{d\theta} \cdot \sin\theta + R \cdot \cos\theta$$

$$= \cos\theta \cdot \sin\theta + R \cdot \cos\theta \xrightarrow[R = 1 + \frac{\sqrt{3}}{2}]{\theta = \frac{\pi}{3}} \frac{\sqrt{3}}{4} + \left(1 + \frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1 + \sqrt{3}}{2}$$

$$\frac{dx}{d\theta} = \frac{dR}{d\theta} \cdot \cos\theta - R \cdot \sin\theta = \cos^2\theta - R \cdot \sin\theta$$

$$\xrightarrow[R = 1 + \frac{\sqrt{3}}{2}]{\theta = \pi/3} \frac{1}{4} - \left(1 + \frac{\sqrt{3}}{2}\right) \cdot \frac{\sqrt{3}}{2} = \frac{-1 - \sqrt{3}}{2}$$

$$\text{Slope} = \frac{dy/d\theta}{dx/d\theta} = \frac{(1 + \sqrt{3})/2}{(-1 - \sqrt{3})/2} = \boxed{-1}$$