Review 3

Tuesday, November 27, 2018 8:00 AM

5.5 (1): 5, 6, 9, 16, 17

5.6 (1): 3, 5, 7, 8, 9, 10

6.1 (2): 1, 3, 7, 9, 17, 19

6.2 (2): 3, 4, 7, 9, 11, 15

6.3 (2): 7, 10, 14, 15, 16, 20

6.4 (2): 1, 2, 3, 5, 8, 9, 12, 13, 14

Sample Solutions

$\boxed{5.6 \ \#3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n}{8^n}$; $a_n = \dfrac{n}{8^n}$. Ratio Test.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{n+1}{8^{n+1}} \cdot \frac{8^n}{n}\right| = \left|\frac{n+1}{8n}\right|$$

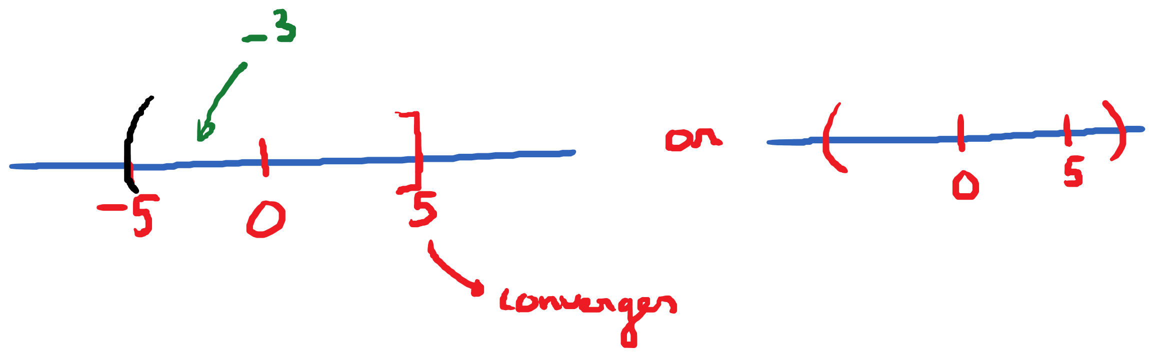
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{n+1}{8n} = \frac{1}{8} < 1$$

So series converges.

$\boxed{6.1 \ \#9}$

Given: $\sum_{n=0}^{\infty} c_n 5^n$ converges. Series: $\sum_{n=0}^{\infty} c_n x^n$

Center $= 0$; converges when $x = 5$



$\sum_{n=0}^{\infty} c_n (-3)^n$ converges or not? $x = -3$. Yes.

$\sum_{n=0}^{\infty} c_n (-5)^n$ converges or not? $x = -5$. Inconclusive

$\boxed{6.3 \ \#7}$ $f(x) = 5x \cos\left(\frac{1}{8}x^2\right)$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!};$$

$$5x\cos\left(\frac{1}{8}x^2\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(\frac{1}{8}x^2\right)^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 5}{(2n)!\,64^n} x^{4n+1}$$

$\boxed{6.2 \ \#3}$ (a) $\boxed{f(x) = \dfrac{1}{(4+x)^2}}$

$$\boxed{\frac{1}{4+x}} = \frac{1}{4\left(1-\left(-\frac{x}{4}\right)\right)} = \frac{1}{4}\sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n \quad ; \ |x| < 4$$

$$= \frac{1}{4}\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{4^n} = \boxed{\sum_{n=0}^{\infty}\frac{(-1)^n}{4^{n+1}}x^n}$$

Differentiate

$$\boxed{\frac{-1}{(4+x)^2}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{4^{n+1}}\cdot n x^{\boxed{n-1}} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{4^{n+2}}(n+1)x^n$$

$$\boxed{\frac{1}{(4+x)^2} = \sum_{n=0}^{\infty}\frac{(-1)^n}{4^{n+2}}(n+1)x^n}$$

Differentiate

$$\boxed{\frac{1}{(4+x)^3}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{4^{n+2}}(n+1)n x^{n-1} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{4^{n+3}}(n+2)x^n$$

$$\rightarrow \frac{x^2}{(4+x)^3} = \cdots\cdots x^{n+2}$$

$\boxed{5.6\ \#7}$

$$\sum_{n=1}^{\infty} \frac{5\cdot 10\cdot 15\cdots(5n)}{n!}$$

$$a_n = \frac{5\cdot 10\cdot 15\cdots(5n)}{n!} = \frac{5^n\cdot n!}{n!} = 5^n$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5^{n+1}}{5^n}\right| = 5. \quad \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 5 > 1$$

Diverges.

$\boxed{6.3\ \#15}$

$$\sum_{n=0}^{\infty} \frac{5(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!}$$

$$5\cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{6}\right)^{2n+1} = 5\cdot \sin\left(\frac{\pi}{6}\right)$$

$$= \boxed{\frac{5}{2}}$$

$$\boxed{\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}}$$

Tuesday, November 27, 2018 8:57 AM

$\boxed{6.3 \ \#14}$

$$\sum_{n=0}^{\infty} (-1)^n \frac{6^n x^{2n}}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (6x^2)^n}{n!}}$$

$$\boxed{e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}}$$

$$\boxed{e^{-6x^2}}$$

$\boxed{6.1\ \#7}$ $\sum_{n=1}^{\infty} n!\,(8x-1)^n$. R.O.C. and I.O.C

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)!\,(8x-1)^{n+1}}{n!\,(8x-1)^n}\right|$$

$$= (n+1)\,|8x-1|$$

$$\lim_{n\to\infty} (n+1)\,\underbrace{|8x-1|} = \infty > 1 \left(\text{unless } x = \frac{1}{8}\right)$$

Series diverges everywhere except for $x = \frac{1}{8}$

$R = 0$; $I = \left\{\frac{1}{8}\right\}$

$\boxed{6.4 \# 8}$

$$f(x) = \frac{\sin(\sqrt{x})}{\sqrt{x}}$$

$$\boxed{\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}}$$

$$\frac{\sin(\sqrt{x})}{\sqrt{x}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^n$$

Tuesday, November 27, 2018 9:30 AM

$$\frac{1}{n+1} \cdot t^{n+1} \Big|_0^x$$

#12

$$F(x) = \int_0^x \cos(\sqrt{t})\,dt$$

$$= \int_0^x \sum_{n=0}^{\infty} (-1)^n \cdot \frac{t^n}{(2n)!}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \boxed{\int_0^x t^n\,dt}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{n+1}}{n+1}$$

6.4 #9

$$\frac{x}{9+x^2} = \frac{x}{9\left(1-\left(-\frac{x^2}{9}\right)\right)} = \frac{1}{9}x \sum_{n=0}^{\infty} \left(-\frac{x^2}{9}\right)^n$$

$$= \frac{1}{9}x \sum_{n=0}^{\infty} \frac{(-1)^n}{9^n} x^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{9^{n+1}} x^{2n+1}$$

$$\left|\frac{x^2}{9}\right| < 1 \;;\; |x|^2 < 9 \;;\; |x| < 3 \;;\; (-3, 3)$$

6.2 # $\frac{1}{x-8} - \frac{1}{x+3} = f(x)$ $|x| < 8$ $\boxed{|x| < 3}$

$$\frac{1}{-8+x} - \frac{1}{3+x} = \frac{1}{-8\left(1-\frac{x}{8}\right)} - \frac{1}{3\left(1+\frac{x}{3}\right)}$$

$$= -\frac{1}{8}\sum_{n=0}^{\infty}\frac{x^n}{8^n} - \frac{1}{3}\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{3^n}.$$

$$= \sum_{n=0}^{\infty}\left[-\frac{1}{8^{n+1}} - \frac{(-1)^n}{3^{n+1}}\right]x^n.$$