$\boxed{\#8}$ $\underbrace{y = \ln(x^7)}_{b}$; $1 \le x \le 7$; Rotate about x-axis.

$y = \ln(x^7) = 7 \ln x$

$$S = 2\pi \int_a^b y \, ds$$

Integrate w.r.t. x. $ds = \sqrt{1 + [f'(x)]^2} \, dx$

$$f'(x) = \frac{7}{x} \rightarrow ds = \sqrt{1 + \left(\frac{7}{x}\right)^2} \, dx$$

$$S = 2\pi \int_1^7 \underbrace{\ln(x^7)}_{y} \sqrt{1 + \left(\frac{7}{x}\right)^2} \, dx$$

$$S = 2\pi \int_1^7 7 \ln(x) \sqrt{1 + \left(\frac{7}{x}\right)^2} \, dx$$

Integrate w.r.t. y.

$$y = 7\ln(x) \longrightarrow \ln(x) = \frac{y}{7} \longrightarrow x = e^{\frac{y}{7}}.$$

$$S = 2\pi \int y \cdot \sqrt{1 + [g'(y)]^2}\, dy$$

$$g'(y) = \frac{dx}{dy} = \frac{1}{7} \cdot e^{\frac{y}{7}}$$

Bounds for y: $x = 1 \rightarrow y = 7\ln(1) = 0$

$x = 7 \rightarrow y = 7\ln(7)$

$$S = 2\pi \int_0^{7\ln(7)} y \cdot \sqrt{1 + \left(\frac{1}{7} e^{y/7}\right)^2}\, dy.$$

$\boxed{\#9}$

$$S = 2\pi \int x \, ds$$

$\rightarrow$ Integrate with respect to x.

$$ds = \sqrt{1 + [f'(x)]^2} \, dx$$

$$f'(x) = \frac{1}{2}x - \frac{1}{2x}$$

$$S = 2\pi \int_1^8 x \cdot \sqrt{1 + \left(\frac{1}{2}x - \frac{1}{2x}\right)^2} \, dx.$$

$$= 2\pi \int_1^8 x \cdot \sqrt{1 + \left(\frac{1}{2}x\right)^2 - \frac{1}{2} + \left(\frac{1}{2x}\right)^2} \, dx$$

$$= 2\pi \cdot \int_1^8 x \cdot \sqrt{\underbrace{\left(\frac{1}{2}x\right)^2}_{A^2} + \underbrace{\frac{1}{2}}_{2 \cdot \frac{1}{2}x \cdot \frac{1}{2x}} + \underbrace{\left(\frac{1}{2x}\right)^2}_{B^2}} \, dx$$

$2AB$

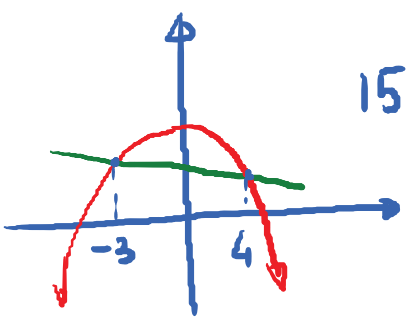
$$= 2\pi \cdot \int_1^8 x \cdot \sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \cdot \int_1^8 x \cdot \left(\frac{1}{2}x + \frac{1}{2x}\right) dx$$

$$= 2\pi \cdot \int_1^8 \left(\frac{1}{2}x^2 + \frac{1}{2}\right) dx = 2\pi \left(\frac{x^3}{6} + \frac{x}{2}\right)\Bigg|_1^8 = \cdots$$

$\boxed{\#10}$

$$\bar{x} = \frac{M_y}{m} \; ; \; \bar{y} = \frac{M_x}{m}$$



$$15 - x^2 = 3 - x \rightarrow x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

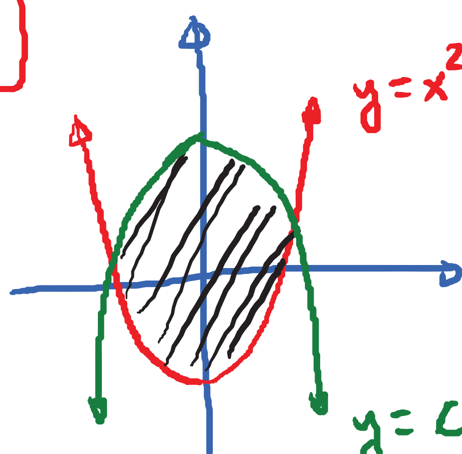
$$m = \int_{-3}^{4} \left[(15 - x^2) - (3 - x)\right] dx = \frac{343}{6}$$

$$M_y = \int_{-3}^{4} x\cdot\left[(15-x^2)-(3-x)\right]dx = \frac{343}{12}$$

$$M_x = \frac{1}{2}\int_{-3}^{4}\left[(15-x^2)^2-(3-x)^2\right]dx = \frac{12691}{30}$$

$$\bar{x} = \frac{343/12}{343/6} = \boxed{\frac{1}{2}} \quad ; \quad \bar{y} = \frac{12691/30}{343/6} = \boxed{\frac{37}{5}}.$$

$\boxed{\#2}$



$$x^2 - c^2 = c^2 - x^2$$

$$2x^2 = 2c^2 \rightarrow x^2 = c^2 \rightarrow x = \pm c.$$

$$\int_{-c}^{c}\left[(c^2-x^2)-(x^2-c^2)\right]dx$$

$$= \int_{-c}^{c}(2c^2-2x^2)\,dx = \left(2c^2x - \frac{2x^3}{3}\right)\Bigg|_{-c}^{c}$$

$$= \left(2c^3 - \frac{2c^3}{3}\right) - \left(-2c^3 + \frac{2c^3}{3}\right)$$

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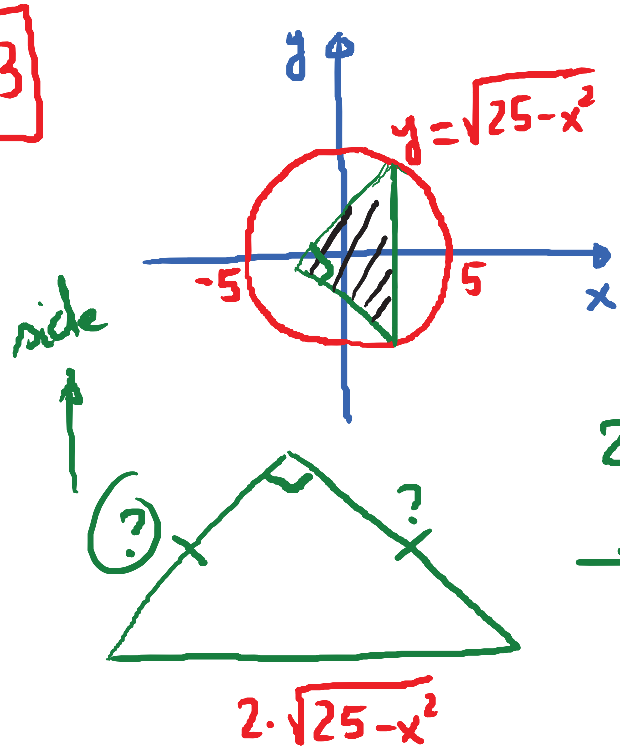
$A = \frac{8c^3}{3}$. Set $\frac{8c^3}{3} = 72 \to c^3 = 27 \to \boxed{c = 3}$.

$\boxed{\#3}$



$\text{Area} = \frac{1}{2}(\text{side})^2$

$\text{Hypotenuse} = 2\cdot\sqrt{25-x^2}$

$2\cdot(?)^2 = 4(25-x^2)$

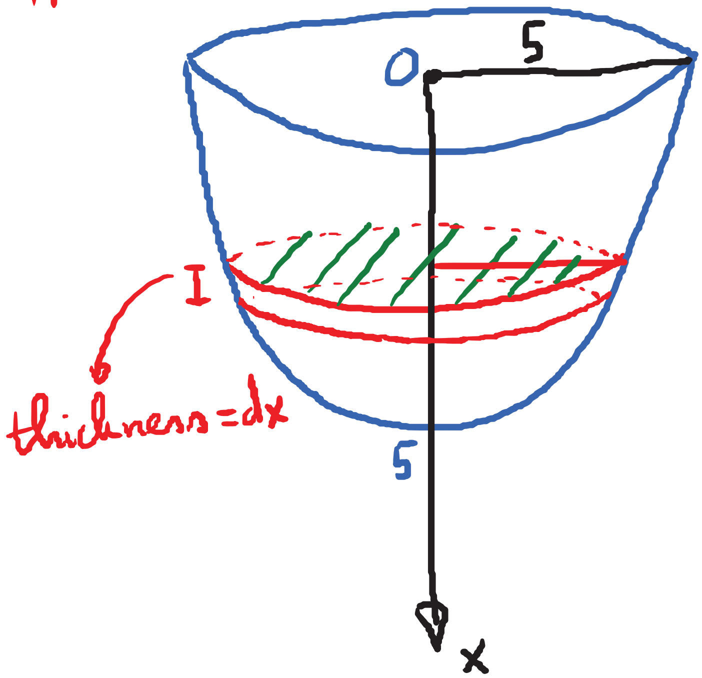
$\to (?)^2 = 2(25-x^2)$

$? = \sqrt{2(25-x^2)}$

$$A(x) = \frac{1}{2}\cdot\left(\sqrt{2(25-x^2)}\right)^2 = 25-x^2$$

$$V = \int_{-5}^{5} A(x)\,dx = \int_{-5}^{5} (25-x^2)\,dx = \cdots$$

#4



weight of slice = (density)·(volume)

$= (62.5)\cdot(\text{base area})\cdot dx$

$= (62.5)\cdot\left(\pi\cdot(\text{radius})^2\right)\cdot dx$

$= (62.5)\cdot\left(\pi\cdot\left(\sqrt{25-x^2}\right)^2\right)dx$

$$\text{Work} = \int_0^5 62.5\pi\left(25-x^2\right)dx = \cdots$$