Test 1 Review.

Tuesday, September 18, 2018 9:56 AM

$\boxed{\#1}$

$$\text{Area} = \int \left(\text{top curve} - \text{bottom curve}\right)$$

$$\text{Area} = \int_0^1 \left[\left(-6x^2+6x\right) - \left(x-\sqrt{x}\right)\right] dx$$

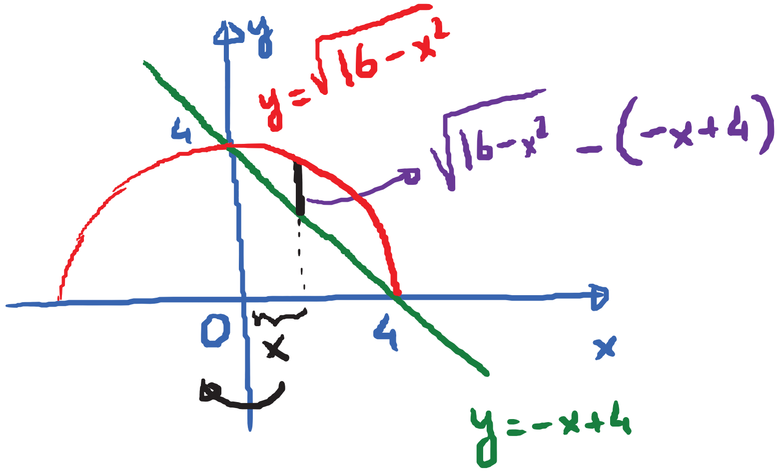
$$= \int_0^1 \left(-6x^2 + 5x + x^{\frac{1}{2}}\right) dx$$

$$= \left(-2x^3 + \frac{5x^2}{2} + \frac{2x^{\frac{3}{2}}}{3}\right)\Bigg|_0^1$$

$$= -2 + \frac{5}{2} + \frac{2}{3} = \boxed{\frac{7}{6}}$$

(TI-84: Math → 9 → type in integral)

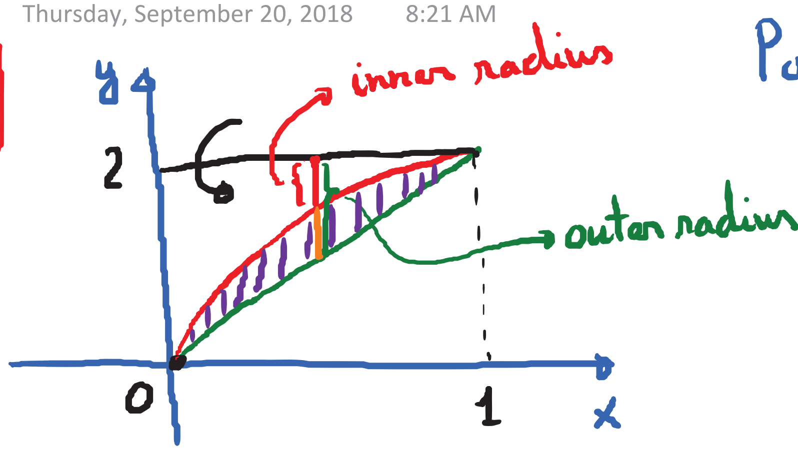
$\boxed{\#2}$ $$V_{\text{shell}} = 2\pi \int_a^b (\text{radius})_{\text{shell}} (\text{height})_{\text{shell}} \cdot (\text{thickness})$$



$$V = 2\pi \int_0^4 x \cdot \left(\sqrt{16-x^2} + x - 4\right) dx$$

$$= 2\pi \left[\underbrace{\int_0^4 x \cdot \sqrt{16-x^2}\, dx}_{\substack{u = 16 - x^2 \\ du = -2x\,dx}} + \underbrace{\int_0^4 (x^2 - 4x)\, dx}_{\text{easy}} \right]$$

$\boxed{\#3}$



Points of Intersection:

$$2\sqrt[4]{x} = 2x$$

$$\sqrt[4]{x} = x$$

$$x = x^4 \rightarrow x(x^3-1)=0$$

$$\rightarrow x=0 \; ; \; x=1$$

Washer: Inner Radius $= 2 - 2\sqrt[4]{x}$

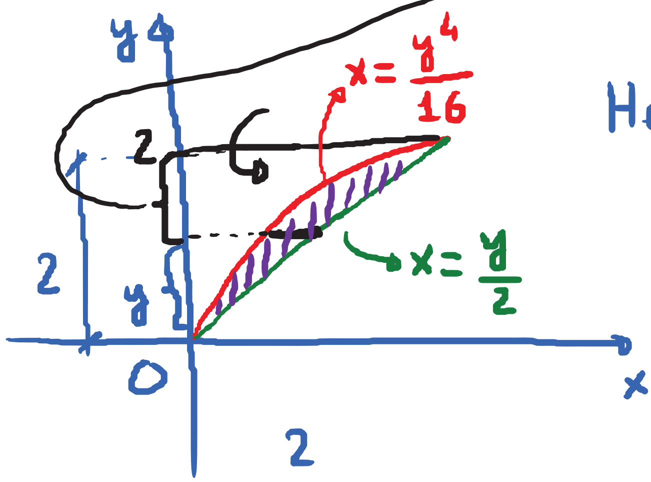
Outer Radius $= 2 - 2x$

$$V = \pi \cdot \int \left[(\text{outer radius})^2 - (\text{inner radius})^2\right](\text{thickness})$$

$$= \pi \int_0^1 \left[(2-2x)^2 - (2-2\sqrt[4]{x})^2\right] dx$$

$$\approx 1.067\pi$$

Shell Method:



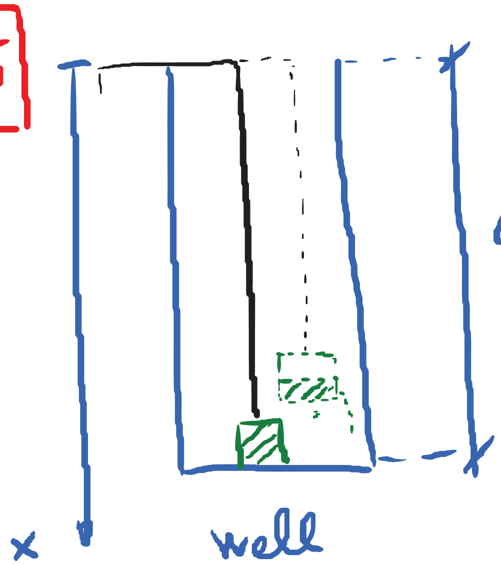
$$\text{Radius}_{shell} = 2 - y$$

$$\text{Height}_{shell} = \frac{y}{2} - \frac{y^4}{16}$$

$$V = 2\pi \int_0^2 (2-y)\left(\frac{y}{2} - \frac{y^4}{16}\right) dy = \cdots$$

$\boxed{\#4}$ Done in 2.5.

$\boxed{\#5}$



$$W = \int_0^{40} \underbrace{F(x)}_{\text{force}}\, dx$$

$F(x)$ = weight of bucket

→ Find a function which models the weight of bucket during the process

Thursday, September 20, 2018 8:45 AM

$x = 40 - 10\boxed{t}$ → this is how the distance is changing w.r.t. time

time

$w = 50 - 0.5t$ → this is how the weight is changing w.r.t. time

$10t = 40 - x \rightarrow t = \frac{40 - x}{10}$

$$w = 50 - 0.5 \cdot \left(\frac{40 - x}{10} \right)$$

this is how the weight is changing w.r.t. distance x

$$\text{Work} = \int_0^{40} \left[50 - 0.5 \left(\frac{40 - x}{10} \right) \right] dx + \int_0^{40} 5\,dx$$

$\boxed{\#6}$

$$\int x^5 \ln x \, dx$$

$$\begin{cases} u = \ln x \\ dv = x^5 \, dx \end{cases} \longrightarrow \begin{cases} du = \dfrac{1}{x} \, dx \\ v = \dfrac{x^6}{6} \end{cases}$$

$$\int x^5 \ln x \, dx = \frac{x^6 \ln x}{6} - \int \frac{x^6}{6} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^6 \ln x}{6} - \frac{1}{6} \int x^5 \, dx$$

$$= \boxed{\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C}$$

$\boxed{\#7}$ Arc length $= \displaystyle\int_a^b \sqrt{1 + [f'(x)]^2} \, dx$

$$y = 2 \ln\left(\sin \frac{x}{2}\right)$$

$$f'(x) = \frac{dy}{dx} = \cancel{2} \cdot \frac{1}{\sin\left(\frac{x}{2}\right)} \cdot \cos\left(\frac{x}{2}\right) \cdot \frac{1}{\cancel{2}}$$

$$= \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} = \cot\left(\frac{x}{2}\right)$$

$$\rightarrow L = \int_{\frac{\pi}{5}}^{\pi} \sqrt{\underbrace{1 + \cot^2\left(\frac{x}{2}\right)}_{\csc^2\left(\frac{x}{2}\right)}}\, dx$$

$$L = \int_{\frac{\pi}{5}}^{\pi} \csc\left(\frac{x}{2}\right) dx = -2\ln\left|\csc\left(\frac{x}{2}\right) + \cot\left(\frac{x}{2}\right)\right| \Bigg|_{\frac{\pi}{5}}^{\pi}$$

<u>Formula:</u> $\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$