Review 2

Thursday, October 25, 2018 8:36 AM

Essay Part

1. $$\int \frac{\sqrt{25x^2 - 16}}{x}\,dx \quad ; \quad x = \frac{4}{5}\sec\theta$$

. . .

2. $$\int \tan^3 x \sec^5 x \, dx$$

$$= \int \tan^2 x \sec^4 x \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1)\sec^4 x \underbrace{\boxed{\tan x \sec x \, dx}}_{du}$$

Let $u = \sec x$; $du = \sec x \tan x \, dx$

$$= \int (u^2 - 1)u^4 \, du \longrightarrow \ldots \text{ easy.}$$

③ $$\int \frac{x^2 - x + 2}{x^3 - 2x^2 + x}\,dx = \int \frac{x^2 - x + 2}{x(x^2 - 2x + 1)}\,dx$$

$$= \int \frac{x^2 - x + 2}{x(x-1)^2}\,dx$$

$$\frac{x^2 - x + 2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

④ $$3\int_{-\infty}^{\infty} \frac{e^x}{3+e^{2x}}\,dx = 3\cdot\left[\int_{-\infty}^{0} \frac{e^x}{3+e^{2x}}\,dx + \int_{0}^{\infty} \frac{e^x}{3+e^{2x}}\,dx\right]$$

$$= 3\cdot\left[\lim_{t\to-\infty} \int_{t}^{0} \frac{e^x}{3+e^{2x}}\,dx + \lim_{t\to\infty} \int_{0}^{t} \frac{e^x}{3+e^{2x}}\,dx\right]$$

$$\int \frac{e^x}{3+e^{2x}}\,dx \quad ; \qquad u = e^x \ ; \qquad du = e^x\,dx$$

du

$$\int \frac{du}{3+u^2} = \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right)$$

$$\frac{1}{\sqrt{3}} \arctan\left(\frac{e^x}{\sqrt{3}}\right)$$

$$3.\left[\lim_{t\to-\infty} \frac{1}{\sqrt{3}} \arctan\left(\frac{e^x}{\sqrt{3}}\right)\Bigg|_t^0 + \lim_{t\to\infty} \frac{1}{\sqrt{3}} \arctan\left(\frac{e^x}{\sqrt{3}}\right)\Bigg|_0^t\right]$$

$$3\left[\frac{1}{\sqrt{3}}\lim_{t\to-\infty}\left(\arctan\left(\frac{1}{\sqrt{3}}\right)-\cancel{\arctan\left(\frac{e^t}{\sqrt{3}}\right)}\right)\right. \to 0$$

$$\left.\frac{1}{\sqrt{3}}\lim_{t\to\infty}\left(\underbrace{\arctan\left(\frac{e^t}{\sqrt{3}}\right)}_{\frac{\pi}{2}}-\arctan\left(\frac{1}{\sqrt{3}}\right)\right)\right]$$

$$3\cdot\left[\cancel{\frac{1}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}\right)}+\frac{1}{\sqrt{3}}\cdot\frac{\pi}{2}-\cancel{\frac{1}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}\right)}\right]$$

$$=3\cdot\frac{1}{\sqrt{3}}\cdot\frac{\pi}{2}=\boxed{\frac{\pi\sqrt{3}}{2}}$$

5

$$a_n = \frac{-2}{(n+7)(n+5)}$$

$$\frac{1}{(n+7)(n+5)} = \frac{A}{n+7} + \frac{B}{n+5} = \frac{-1/2}{n+7} + \frac{1/2}{n+5}$$

$$a_n = \frac{1}{n+7} - \frac{1}{n+5}.$$

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty}\left[\frac{1}{n+7} - \frac{1}{n+5}\right]$$

$$s_m = \sum_{n=0}^{m}\left[\frac{1}{n+7} - \frac{1}{n+5}\right]$$

$$= \left(\cancel{\frac{1}{7}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{8}} - \frac{1}{6}\right) + \left(\frac{1}{9} - \cancel{\frac{1}{7}}\right)$$

$$+ \left(\cancel{\frac{1}{10}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{12}} - \cancel{\frac{1}{10}}\right) + \cdots \left(\frac{1}{m+7} - \frac{1}{m+5}\right)$$

$$\lim_{m\to\infty} s_m = -\frac{1}{5} - \frac{1}{6} + \frac{1}{9} =$$

6.

$$\sum_{m=2}^{\infty} \frac{2}{m(\ln m)^2}$$

Function associated with this series is

$$f(x) = \frac{2}{x(\ln x)^2}$$

For sum to be ≤ 0.02, we want upper bound of error to be ≤ 0.02.

$$\int_N^{\infty} \frac{2}{x(\ln x)^2}\,dx \leq 0.02$$

$$u = \ln x \;;\; du = \frac{1}{x}\,dx$$

$$\int \frac{du}{u^2} = -\frac{1}{u}$$

$$-\frac{2}{\ln(x)}\Bigg|_N^{\infty} = \frac{2}{\ln(N)}$$

So,

$$\frac{2}{\ln(N)} \leq 0.02 \rightarrow 2 \leq (0.02)\ln(N)$$

$$\ln(N) \geq 100 \rightarrow N \geq e^{100}$$

7) Since $\sin(6n) \geq -1$

$$3 + \sin(6n) \geq 2$$

So, the series does have positive terms.

$$\sin(6n) \leq 1$$

$$3 + \sin(6n) \leq 4$$

$$\frac{3+\sin(6n)}{4^n} \leq \frac{4}{4^n} \text{ ; for all } n \geq 1$$

$$\sum_{n=1}^{\infty} \frac{3+\sin(6n)}{4^n} \leq \boxed{\sum_{n=1}^{\infty} \frac{4}{4^n}}$$

geometric, common ratio $= \frac{1}{4}$. Hence, it converges

By the comparison test, the series converges.

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1. $a^{\ln(b)} = b^{\ln(a)}$

$$\sum_{m=1}^{\infty} 6u^{\ln(7m)} = 6\sum_{m=1}^{\infty} u^{\ln 7 + \ln m}$$

$$= 6\sum_{m=1}^{\infty} u^{\ln 7} \cdot u^{\ln m}$$

$$= 6u^{\ln 7} \sum_{m=1}^{\infty} u^{\ln m} = 6u^{\ln 7} \cdot \sum_{m=1}^{\infty} m^{\boxed{\ln u}}$$

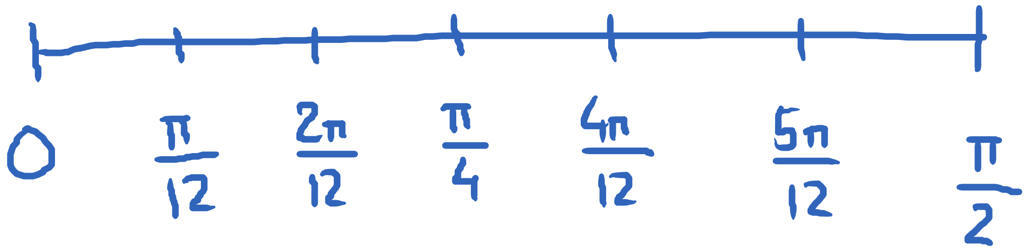
$$= 6u^{\ln 7} \cdot \sum_{m=1}^{\infty} \left(\frac{1}{m}\right)^{-\ln u} \qquad -\ln u \rightarrow p$$

$p > 1 \rightarrow -\ln u > 1 \rightarrow \ln u < -1$

$u < e^{-1} = \frac{1}{e}$.

$$\boxed{0 < u < \frac{1}{e}}$$

1



$$\Delta x = \frac{\pi}{12} \; ; \; f(x) = \sqrt{1 + \sin^2 x} \, .$$

$$S_6 = \frac{\pi}{36}\Big[f(0) + 4f\left(\tfrac{\pi}{12}\right) + 2f\left(\tfrac{\pi}{6}\right) + 4f\left(\tfrac{\pi}{4}\right) + 2f\left(\tfrac{\pi}{3}\right) + 4f\left(\tfrac{5\pi}{12}\right) + f\left(\tfrac{\pi}{2}\right) \Big]$$